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Analysis of the 2025 Bundestag elections. Part 3 of 4: The third vote perspective Bundestag

by Andranik S. Tangian

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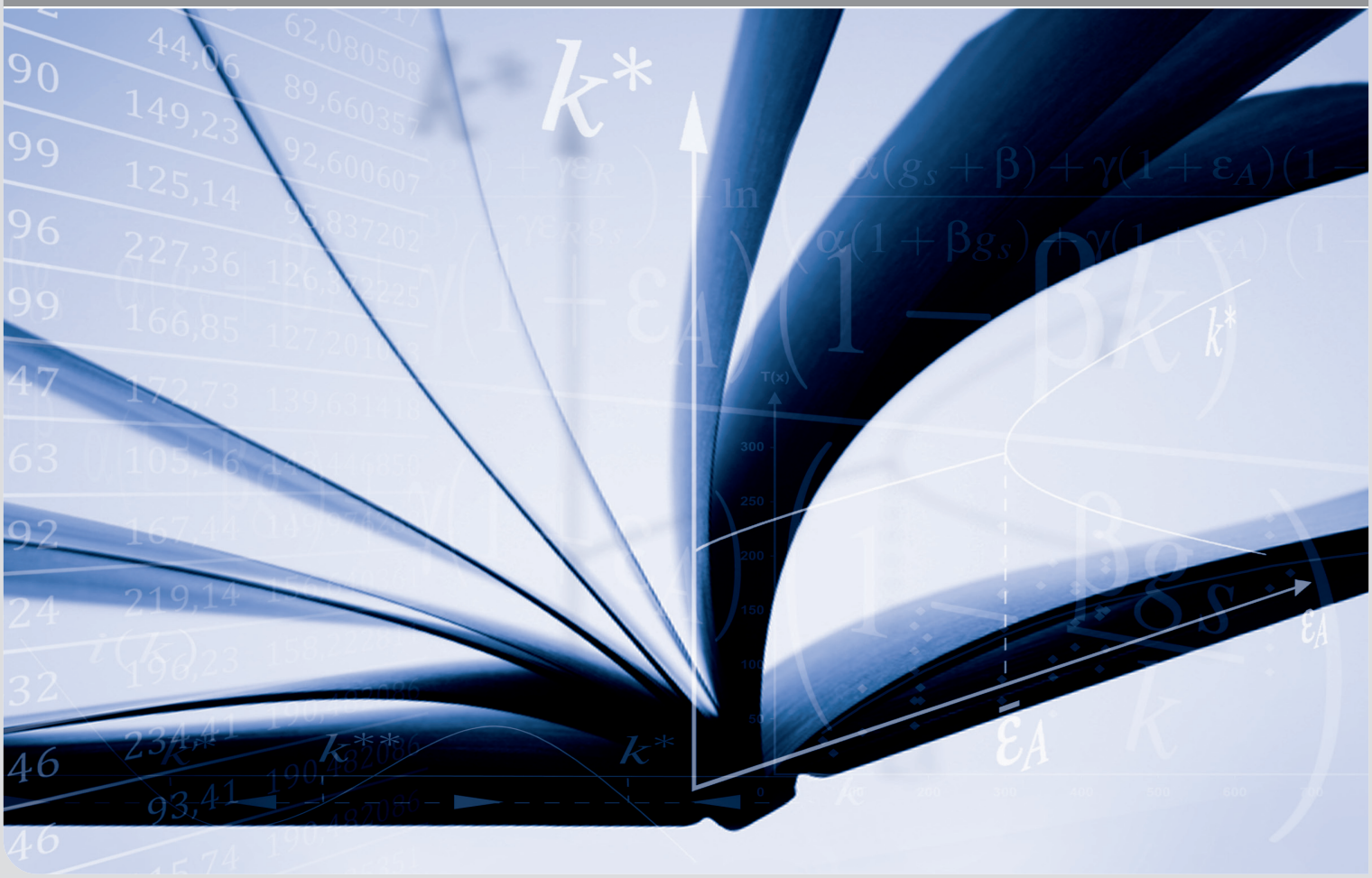

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Analysis of the 2025 Bundestag Elections. 3/4. The Third Vote Perspective

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Abstract

This is the third out four papers on the 2025 German Bundestag elections continuing our analysis of the 2009, 2013, 2017 and 2021 elections. In particular, this paper contributes to the discussion of the imperfection of the German 2023/24 Electoral reform in [Tangian 2025a].

We show that policy representation by the Bundestag could be improved using the alternative Third Vote election method. Under the Third Vote, electors cast no votes for parties by name. The electoral ballot consists of questions on topical policy issues ('General speed limit on motorways?'—Yes/No, 'Germany should increase its defense spending?'—Yes/No, etc.), and the parties answer to these questions before the elections — as required by the *Wahl-O-Mat*, the German voting advice applications. However, the Third Vote is not concerned with individual advices or individual voting intermediation. The electoral ballots are processed to construct the electorate's policy profile with balances of public opinion on all the issues. Then the matching of the parties' profiles with the electorate's profile is measured using the parties' indices of popularity (average percentage of electors represented on all the issues) and universality (percentage of questions when a majority is represented). These indices of representativeness are used instead of the conventional index 'number of votes received' to define the party quotas in the Bundestag. This method is hypothetically applied to reallocate the 2025 Bundestag seats to the eligible parties, resulting in a considerable gain in the Bundestag representativeness. Finally, we discuss mixed election procedures combining the Third Vote with the conventional voting by party name and analyze possible implications.

Keywords: Representative democracy, elections, theory of voting, proportional representation.

JEL Classification: D71

Contents

1 Introduction

This is the third out four papers on the 2025 German Bundestag elections continuing our analysis of the elections in 2009, 2013, 2017 and 2021 [Tangian 2014, 2020, 2022a–d]. In particular, this paper contributes to the discussion of the imperfection of the German 2023/24 Electoral reform in [Tangian 2025a]. The structure of the paper follows [Tangian 2022d], from which we quote for the reader's convenience without special reference.

Under representative democracy, participation by the people — the main purpose of democracy — is realized through the election of representatives. The representatives are empowered, on behalf of the people, to make political decisions and supervise vital attributes of democracy such as the rule of law, human rights, and freedom of the press. Therefore, representative democracy is only truly *democratic* if the elected actually *represent* the public interest. Correspondingly, the main criterion of democratic performance is the degree of representativeness of the governing institutions.

In [Tangian 2020, Section 5.3], two major historical concepts of political representation are discussed: one that is descriptive, requiring representatives to be local delegates and leading to proportional representation, and an agent one, allowing representatives to be external trustees and leading to majoritarianism. These concepts are implemented in different electoral systems and different methods of allocating parliamentary seats. The German electoral system with its two simultaneous votes combines both. In the 2021 federal elections, 299 Bundestag seats were given to 'direct mandate holders' — the deputies from 299 constituencies elected locally by simple plurality by the first vote (according to the descriptive concept). The next 299 regular Bundestag seats and some leveling seats were distributed among eligible parties to adjust their factions to the party quotas in proportions of the second votes cast nationwide (according to the agent concept). According to the 2023/24 electoral reform, the size of the Bundestag is now fixed at 630 seats, and not all constituency election winners are awarded with Bundestag mandates; the limits are determined by the results of the second vote within federal states. In the 2025 Bundestag elections, this restriction left 23 constituency election winners without Bundestag seats; for details and analysis see [Tangian 2025a]. Thereby, the reform shifted the German electoral system more toward the agent concept.

At the same time, the fundamental question, discussed since Rousseau's *Social Contract*, remains open: how, exactly, is political representation related to government by the people, or democracy? The related problems were realized in the 1960s when academics introduced the concept of *policy representation* to monitor the quality of political intermediation — how well the party system and the government represent policy preferences of the electorate [Miller and Stokes 1963, Pitkin 1967]; for a brief survey see [Tangian 2017a]. The concept of policy representation complements the descriptive and agent concepts in the following way: the descriptive concept is concerned with the question *'Who is represented?'*; the agent concept with *'Who are representatives?'*; but both disregard the question *'What is represented?'* — which is the subject of policy representation.

It is not surprising that policy representation was missing from the debates of the 18th century, when the idea of representation was coined. The acute questions of that time were voting rights, property qualifications, gender restrictions, assembly composition, etc., that is, who should vote (who is represented), and eligibility conditions for public offices, that is, who can be elected and to which offices (who are representatives). The uneducated common folk had no policy preferences on most policy issues (with a few exceptions for voting rights, equality before the law or taxation); the mass-communication possibilities were limited, and politicians seldom, if ever, made their positions on every issue public. Since political decisions were entrusted to elected representatives, government by the people was not on the agenda; furthermore, the Founding Fathers of the United States were explicitly against democracy [Manin 1997, Ch. 3]. Correspondingly, neither the American Constitution nor any Amendment to it ever refers to democracy; democracy is also completely missing from the French post-revolutionary constitutions; see [Tangian 2020, Section 5.7].

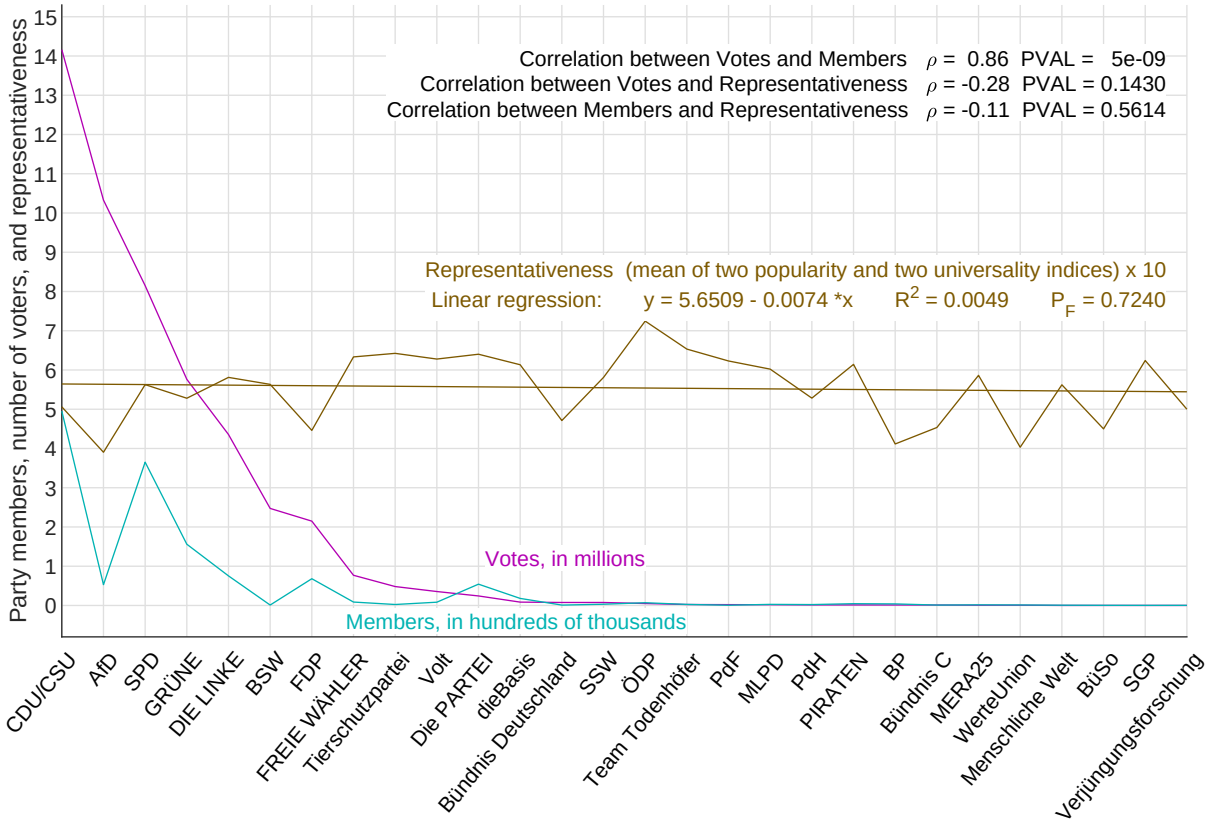

Figure 1: Size of German parties at the 2025 Bundestag election, votes received, and the mean representativeness index

Now the situation is different. The voting rights and eligibility for offices are no longer disputed, because all restrictions for citizens are removed. The population has become more advanced politically, mass media make information available, and politicians' manifestos are publicly discussed. Policy representation is becoming more important as a bridge between representative government and government by the people. However, candidates' positions and party manifestos receive little attention from the electorate. The question *'What is represented?'* is still insufficiently elucidated, and the existing electoral systems bear some of the blame for that.

Figure 1 illustrates the coordination between the three aspects (concepts) of representation. The question *'Who is represented?'* is depicted by the curve 'Votes', because the represented are voter groups who cast their votes for this or that party. The question *'Who are representatives?'* is visualized by the curve 'Members', for the size of the parties that represent the corresponding voter groups, i.e. the representatives are parties. The question *'What is represented?'* is associated with the curve 'Representativeness', because the matters to be represented are the people's policy preferences, whose representation quality is characterized by the mean of the parties' popularity and universality indices defined and computed in [Tangian 2025b].

As one can see from Figure 1, the first two aspects of representation, 'Votes' and 'Members', which were carefully deliberated by the founders of representative government, are in a good agreement with each other — the larger the voter group, the larger its party-representative. This follows from the high correlation of 0.86 between the curves 'Votes' and 'Members' with the statistical significance PVAL of order 10^{-9}. The causality here is likely two-way. On the one hand, large parties with proven leadership enjoy established reputations, have large networks, raise considerable funds for electoral campaigns and are constantly presenting themselves in the media. On the other hand, the electoral success itself attracts

members through political inclusion, participation in policy making and by enhancing personal influence and career prospects.

At the same time, Figure 1 demonstrates no good coordination between the first two aspects of representation and the third one, policy representation, which was not foreseen in the initial design of representative government. This is visualized by the irregular behavior of the curve 'Representativeness', as opposed to that of the curves 'Votes' and 'Members'. As indicated in the text description at the top of the figure, the correlation of 'Representativeness' with 'Votes' and 'Members' is even negative, being however statistically not significant,[1] meaning that the mutual understanding even between large parties and their numerous adherents is not extendable to the whole of the electorate. Indeed, good coordination between the first two aspects of representation is merely inherent in the near-party circles. The questions *'Who is represented?'* and *'Who are representatives?'*, which single out the groups of party adherents and party members, are not directly addressed to all the people. The 28 parties considered here (the union of CDU and CSU is regarded as one party) express primarily the policy preferences of ca. 1.23 million party members [Niedermayer 2018, p. 6] (see also [List of political parties in Germany 2025]) and not necessarily that of the 60.5 million of eligible German voters [Bundeswahlleiterin 2025, p. 8]. We have here what [Dahl 1989] calls *polyarchy* — rule by the many, but obviously not by the people: even large parties that win elections can fail on representing public opinion. This is generally inherent in one-party systems, and, as we see, this is also true for Germany.

Thus, the answer to the question 'What is most important for a party's electoral success?' can only be: the party's size. This empirical evidence is not what one would expect from representative democracy, indicating a serious gap in its implementation. It seems that elections with voting by party name are designed for two historical concepts of representation — descriptive and agent — but not for policy representation. Indeed, the existing election methods divert the voters' attention away from specific policies, focusing on personalities, ideological labels and slogans.

To counterbalance this bias, the concept of policy representation was (unknowingly) implemented in the pioneering Dutch VAA (voting advice application) *StemWijzer* (= VoteMatch), which was at first distributed on diskettes and in 1998 launched on the internet [ProDemos 2025]; since then, the concept has been appropriated by several other countries [Garzia and Marschall 2014, Garzia and Marschall 2016]. In particular, the German voting advice application we often refer to, the *Wahl-O-Mat* (portmanteau of *Wahl* = election and automate), was introduced in 2002 [Bundeszentrale für politische Bildung 2025]. VAAs ask the user to answer dichotomous policy questions (Reduce immigration?—Yes/No; Increase the defense spending?—Yes/No, etc.) and advise the party that best fits to the user's policy profile, involving thereby the idea of policy representation into play.

Since the two historical types of representation are backed up by their own election methods, it is natural to enhance the third type of representation, policy representation, by designing a dedicated election procedure. In this paper we discuss the *Third Vote* — an election method based on VAA-like questionnaires but operationally different [Tangian 2014, Tangian 2017b, Tangian 2020, Tangian 2022d]. Under the Third Vote, electors cast no votes for parties by name. The electoral ballot consists of VAA-like questions on topical policy issues, and the parties answer to these questions before the elections. However, the Third Vote is not concerned with individual advices or individual voting intermediation. The electoral ballots are processed to construct the electorate' policy profile with the balances of public opinion on every issue. Then the matching of the parties' profiles with the electorate's profile is measured by the parties' indices of popularity (average percentage of electors represented on all the issues) and universality (percentage of cases when a majority is represented). These indices of representativeness are used instead of the conventional index 'number of votes received' to determine the party quotas in the parliament.

To be specific, this method is hypothetically applied to redistribute the Bundestag seats among the party

[1] Figure 1 shows Pearson correlations for party votes in absolute figures, which is different from [Tangian 2025b, Table 4] which shows Spearman rank correlations.

factions in proportion to their indices, producing a significant gain in the representativeness of the Bundestag. Finally, we discuss mixed election procedures combining the elements of conventional voting schemes with the Third Vote.

In Section 2 'Two architectures of election procedure and their political philosophy', two ways of collecting and processing electoral data are compared and their philosophy is discussed.

In Section 3, 'Three concepts of representation — three votes', an alternative election method to enhance policy representation, the Third Vote, is hypothetically applied to redistribute the 2025 Bundestag seats.

Section 4, 'Faction equalization effect and the Third Vote+', explains why the Third Vote tends to equalize the party quotas and why it is not harmful. If desired, a more habitual quota ratio can be obtained using a modification of the Third Vote — the Third Vote+ method.

Section 5, 'Combining the Third Vote with conventional election methods', describes how the first and second votes can be used together with the Third Vote, thereby implementing in elections the concepts of descriptive, agent and policy representation.

Section 6, 'Coalitions in the 2025 Bundestag under the Third Vote', examines the impact of the Third Vote on coalition building.

In Section 7, 'Summary: Enhancing policy representation', the main findings are recapitulated and put into context.

2 Two architectures of election procedure and their political philosophy

Taking into account the adduced reasons, the goal of the Third Vote election method is to redirect the electorate's attention away from candidates as personalities and parties as ideological symbols toward their specific abilities to speak on behalf of the public and represent public opinion on various policy issues. In other words, the question '*Whom* are we electing?' is to be replaced by '*What* are we electing?' If a certain decision, like Brexit, is voted on, then the attention is redirected from the emotional toward rational aspects characterized by specific policy implications.

Before we go on, some remarks should be made on the *architecture* of election procedures — the word is used by analogy with 'software architecture' in computer science, i.e. the way and the order in which information is processed [Tangian 2003, p. 37]. Let us illustrate alternative architectures and their impact on the election outcome using two examples, going back to Ostrogorski's paradox [Nurmi 1999, pp. 70–73]; see also [Gehrlein and Lepelley 2011, pp. 123–124].

Example 1 (Two Architectures of Data Processing in Brexit Referendum) *Brexit — a portmanteau of 'British' and 'exit' for the withdrawal of the United Kingdom from the European Union — was put on a 2016 referendum, in which 51.9% voted to leave. According to the mass media, Brexit has three equally important effects: immigration restriction, closing the Irish border and economic recession/pound devaluation. These policy implications, at first overlooked by most people, were realized only as the British government failed to agree an acceptable Brexit scenario. The society was polarized and the government wallowed in endless debates and negotiations with the European Union. All of these led to a prolonged political crisis [Brexit 2019].*

Let us suppose that Table 1 describes the tolerance for the three Brexit effects in three equally large groups. The first group is negative regarding all the three consequences, but the second and third groups appreciate immigration restriction but differ in their attitudes toward other issues.

The individual determination is implemented in the conventional vote: each individual aggregates his/her opinion profile and makes his/her choice for or against Brexit. The corresponding election architecture has the order of operations $\downarrow\downarrow\downarrow \rightarrow$. Then two of the three groups vote for Brexit (+), and Brexit is approved (+).

Under the public determination, the balances of public opinion on each issue are determined. The

Table 1: Two architectures of voting for Brexit/No Brexit

Brexit implications	Tolerance profiles				Majority vote	Public profile
	1	2	3			
Immigration restriction	$-$	$+$	$+$	$\rightarrow$		$+$
Closing the Irish border	$-$	$+$	$-$	$\rightarrow$		$-$
Economic recession/Pound devaluation	$-$	$-$	$+$	$\rightarrow$		$-$
	$\downarrow$	$\downarrow$	$\downarrow$			$\downarrow$
Votes for Brexit	$-$	$+$	$+$	$\rightarrow$	$+$	
Public attitude toward Brexit						$-$

corresponding election architecture assumes the order of operations $\rightrightarrows\downarrow$. *The resulting public profile consists of public attitudes toward each policy implication: one positive and two negative. After the aggregation of the public profile, Brexit is rejected* $(-)$.

Thus, we can imagine that the Brexit crisis might have been avoided if the Brexit referendum were based on the policy representation concept instead of the conventional Yes/No-vote.

Now we consider a more complex example, illustrating how the election architecture can influence the choice among three candidates.

Example 2 (Two Architectures of Electoral Data Processing for Three Candidates) *Let three candidates for president, A, B and C, and three equal elector groups, 1, 2 and 3, position themselves on three questions: Reform the health care system?—Yes/No, Create more public jobs?—Yes/No, and Increase corporate taxes?—Yes/No as displayed in Table 2. The match-up of candidate positions with the electoral groups is displayed in Table 3. For instance, on Question 1, A represents the opinion of Elector groups 1 and 2, and the opposite opinion of Elector group 3 is represented by B and C. Hence, regarding Question 1, A is supported by 2/3 of the electorate. If the first architecture with the order of operations* $\downarrow\downarrow\downarrow\rightarrow$ *is applied, then each electoral group makes its own choice and casts votes for its favorite candidate. Then the Elector group 1 votes for candidate A, because A represents its opinion on three out of three issues, whereas C represents it on only one issue, and B on no issue at all. The other two electoral groups vote for B, who represents their opinion on two issues, whereas A and C represent them on one issue each. Under this architecture, B wins with 2/3 of the votes. This is the functionality implemented in VAAs (voting advice applications), e.g. Wahl-O-Mat, which finds the party with the closest political profile to that of the user [Bundeszentrale für politische Bildung 2025]. Here, each elector votes as if having been advised by a VAA.*

If the second architecture with the order of operations $\rightrightarrows\downarrow$ *is applied, then the representative of the prevailing public opinion is found for each issue. The first two rows of the table say that, on the first two issues, candidate A represents 2/3 of electors, whereas candidates B and C represent the remaining 1/3 of them. On the third issue, about corporate taxes, A and C represent 1/3 of electors, whereas B represents 2/3 of the electors. Then A is elected, because A represents the prevailing public opinion on two out of three issues, B represents it on one issue, and C on no issue. This architecture, differing from that of VAAs, is used in our indices of popularity and universality. Indeed, as follows from Table 2, the electorate's majority opinions on the issues constitute the vector*

$$\vec{a} = \begin{pmatrix} + \\ + \\ - \end{pmatrix},$$

and the frequencies with which candidates represent majority opinions (= the candidates' universality indices) are equal to $\mathsf{U}_A = 2/3,\ \mathsf{U}_B = 1/3,\ \mathsf{U}_C = 0$. *Hence, candidate A, with the highest universality, is the public choice.*

Table 2: Positions of elector groups 1, 2, 3 and candidates A, B, C on three issues

Question	Elector group's profiles			Candidate's profiles		
	1	2	3	A	B	C
1. Reform the health care system?	+	+	−	+	−	−
2. Create more public jobs?	+	−	+	+	−	−
3. Increase corporate taxes?	+	−	−	+	−	+

Table 3: Two architectures of election procedure for candidates A, B and C

Question	Matching candidates with elector groups				Majority	Public
	1	2	3		vote	match-up
1. Reform the health care system?	A	A	B,C	$\rightarrow$		A
2. Create more public jobs?	A	B,C	A	$\rightarrow$		A
3. Increase corporate tax?	A,C	B	B	$\rightarrow$		B
	$\downarrow$	$\downarrow$	$\downarrow$			$\downarrow$
Electors' votes	A	B	B	$\rightarrow$	B	
Public choice						A

The two collective choice architectures illustrated using these examples are backed up by very different political philosophies. The first architecture denoted by $\downarrow\downarrow\downarrow\rightarrow$ reflects the liberal philosophy of *individual determination*. It is based on individualism in opinions and on understanding the public good as the sum of the good of every individual, in the spirit of John Locke:

> Every man has a 'property' in his own 'person.' This nobody has any right to but himself...
>
> The public good, i.e. the good of every particular member of that society.
>
> [Locke 1689, *Second Treatise of Government*, Ch 5, 26, and *First Treatise of Government*, Ch 9, 92]

This philosophy deals with the aggregation of what Rousseau and Condorcet called individual wills. Electors choose their favorite candidates themselves, according to their own criteria and without being asked why they cast votes for this or that candidate. They can be motivated by personal sympathies or by egoistic intentions rather than by the public good. Even if electors are given some reference criteria, as in our examples, they are free to ignore them or assign arbitrary weights. The privacy in expressing individual wills is incorporated in the electors' votes which are considered 'black boxes'.

The second architecture with the order of operations $\rightrightarrows\downarrow$ reflects the philosophy of *public determination*. It explicitly articulates the public interest, formulating socially important questions and asking for the electors' opinions on them. The society is considered a single body that has a political profile regarding these issues, resembling the Rousseauvian *general will*:

> There is often a great deal of difference between the will of all and the general will; the latter considers only the common interest, while the former takes private interest into account, and is no more than a sum of particular wills...
> [Rousseau 1762, *Of Social Contract*, Book II, 3]

The public political profile is used to find the most socially adequate candidate. This architecture enhances the civic aspect of election and reduces the partiality of electors' opinions. It is often used for evaluating new products, project proposals, scientific contributions, etc., when each referee estimates

each quality separately. For instance, journal articles can be judged by the degree of innovation, awareness of recent literature, and presentation style, with the final decision being based on the collective evaluation of each quality rather than on referee votes.

The two architectures resemble two ways of aggregation in multiple elections [Brams et al. 1998]. Multiple election is selection from bundles of accepted/rejected propositions, e.g. (Create new jobs, Reduce student fees), (Create no new jobs, Reduce student fees), etc. Under the *combination aggregation*, which resembles the first architecture, each elector votes for one of the bundles, and the bundles are ranked by the votes received. In other words, each elector votes for a choice option — composite alternative, controversial candidate, etc. Under the *proposition aggregation*, which resembles the second architecture, each proposition is voted on separately, and the desired bundle is determined. If it is not available, the one that is closest to it is selected. In other words, electors vote for components of the alternatives/characteristrics of the candidates, then the combination of most desired components/characteristics is taken as a reference, and the closest among the available alternatives is selected.

Currently, the mathematical problem of how to best satisfy a collective with a composite program is studied within a new branch of social choice theory called *judgment aggregation* [Grossi and Pigozzi 2014, Lang et al. 2017, List 2012, List and Puppe 2009]. In a sense, our approach is in line with it, having relevance to some other fields as well. For instance, the voters' Yes/No answers to a question imply choosing several candidates (parties) at a time regarding the given issue — it is clearly seen from Example 2 as well as from [Tangian 2025b, Figure 1]. Thereby, the operation on every question of the second architecture resembles *approval voting*, where electors are allowed to vote for all acceptable candidates [Brams and Fishburn 1982, Laslier and Sanver 2010].

Finally, it should be noted that the practice of applying voting wherever possible results in the predominant use of the first architecture and the frequent overlooking of the second one, even in cases where it could be more relevant. For instance, the Condorcet winner is found by means of voting procedures, which embody the philosophy of individual determination, but the winner is misinterpreted as the best from the viewpoint of the general will. The individualistic approach is also seen in the definition of social utility as the sum of individual utilities, although it is not necessarily adequate. Probably, in both cases the second (public) architecture would be more relevant.

3 Three concepts of representation — three votes

The conventional vote by name uses the first election architecture, reflecting the philosophy of individual determination. The second architecture which operationalizes the philosophy of public determination can be implemented as well. The aims are: (a) redirecting the voters' attention from candidate (party) images to their manifestos and policy proposals, and (b) enhancing policy representation. The latter is attained by matching the candidate's policy profiles with that of the electorate. To be specific, we hypothetically modify the German Bundestag election procedure.

We will first focus on the calculation of fractional party quotas for the Bundestag, and then turn to the distribution of the integer-valued number of its seats among the parties in proportion to the quotas.

3.1 Adding a third vote to German ballots

The German two-vote electoral system embodies two major historical concepts of political representation. The descriptive concept — the parliament portrays the society in miniature — is realized by the first vote *(Erststimme)*, with which local candidates are elected within constituencies and delegated to the federal parliament (Bundestag); for details see Introduction, in particular the footnote in p. 1.

The agent concept — the parliament is an assembly of political experts who are the people's trustees and not necessarily their countrymen or fellows — is embodied in the second vote *(Zweitstimme)* for a party. The second vote has two functions: (1) qualifying parties to have the Bundestag seats — those

who receive nationwide at least 5% of the second votes,[2] and (2) determining the parties' shares of the Bundestag — in proportion to the second votes, including the direct mandate holders. For this purpose, the Bundestag seats remaining after the allocation of direct mandate holders are distributed among the eligible parties to make the total of 630 seats.[3] Obviously, the second vote is decisive, because it determines the size of the party factions.

As mentioned in Introduction, the concept of *policy representation* is already implemented in VAAs but they are not electoral but advising devices. Moreover they are addressed to individuals, and the implementation of policy representation concept at the level of the whole of electorate remains beyond their scope. The Third Vote which uses the VAA-like questionnaire fills in this gap. Its versions and implications are discussed in the next sections.

To summarize, the first vote, as before, is cast by name for the Bundestag member from the local constituency. The second vote — for a party — is used to reject unpopular or untrustworthy parties who receive fewer than 5% of the votes. Then the second vote retains only the filtering function, and its second function — determining the party weights in the Bundestag — is conveyed to the third vote. As we see, the partiality of the first vote for a person is reduced by defining quotas in the Bundestag according to the more impersonal second vote for a party. This logic of increasing impartiality of votes is continued in the absolutely impartial third vote.

The parties' policy profiles are backed up by certain ideologies, making the answers to different questions strongly interdependent. Therefore, just a few questions can provide a reliable basis for specifying the political profiles of both parties and electors. In a sense, we speak of a 'direct democracy test' — a competitive public examination of the parties, which are evaluated through a special election procedure with embedded referenda. As usual, the 'examination' can be based on a few key questions that suffice to make a general judgement.

Combining elections with referenda is practiced in Switzerland, Canada, the United States and some other countries, where it is done to avoid multiple campaigns for the convenience of the population [Referendum 2025]. Basing election on referenda is a step further: not only to let the electorate vote on particular actions of the authorities, but to form these authoritative bodies with respect to public opinion and general political context. When representatives are examined using referenda, the electorate gains more control over policy making.

3.2 Advised Vote for individual determination

The first (individualistic) election architecture, shown in Table 3 as $\downarrow\downarrow\downarrow \rightarrow$, assumes processing each individual ballot, whereupon the best matching party receives a vote. The election looks as operating the *Wahl-O-Mat* (or any other VAA), which finds the optimal party for every elector and casts the corresponding vote itself. This mode simulates the behavior of an individual who unconditionally trusts the VAA and follows its advice slavishly. Under this architecture, the election procedure remains traditional: votes are cast for parties and they are tallied as usual. Since this election method is intermediated by the VAAs' operation, it is called *Advised Vote*.

It should be noted that this election method, when applied to choosing the best candidate (party), inherits all paradoxes of voting. Indeed, it assumes the aggregation of numerous preferences and, in particular, can result in the election of the least preferable candidate (Borda paradox) or cyclic majorities (Condorcet paradox). Nevertheless, this method makes sense. Unlike just voting for a seemingly credible candidate,

[2] A party which has at least three direct mandates or represents an ethnical minority is also eligible for Bundestag seats even if it has less than 5% of the second votes.

[3] The Bundestag seats are allocated to parties using the Sainte-Laguë/Scheper method; for details see [Tangian 2025c]. In the 2025 Bundestag elections, the greatest mismatch was between the CSU's 47 constituency election winners (direct mandate candidates) and its too-small 6.92%-quota in the Bundestag [Bundeswahlleiterin 2025, Tangian 2025a]. To adjusted these excessive mandates, the Bundestag would require, even with the tolerated 0.5-seat accuracy, at least $46.5/0.0692 \approx 672$ seats.

Table 4: The 2025 indices of the Bundestag parties, the Bundestag and the hypothetical Bundestag as if elected using Third Vote methods

	P		U		Max quota ratio	1st vote (direct mandates)		2nd vote (by party name)		3rd vote $\frac{P+U}{4}$		3rd vote+ $\frac{P+U}{4}-\frac{1}{2}$		1st vote & 3rd vote	1st vote & 3rd vote+	2nd vote & 3rd vote	2nd vote & 3rd vote+
	u	g	u	g		A	N	A	N	A	N	A	N	N	N	N	N
	%	%	%	%			%	%	%	%	%	%	%	%	%	%	%
CDU/CSU	49	49	53	52		172	62	29	33	51	16	1	2	39	32	25	18
AfD	41	38	42	35		42	15	21	24	39	12	−11	0	14	8	18	12
SPD	54	58	53	60		44	16	16	19	56	18	6	24	17	20	18	22
GRÜNE	53	56	47	55		12	4	12	13	53	17	3	11	11	8	15	12
DIE LINKE	57	60	54	61		6	2	9	10	58	18	8	31	10	17	14	21
SSW	56	60	54	62		0	0	0	0	58	18	8	31	9	16	9	16
Absolute Maximum	71	71	100	100													
BUNDESTAG by																	
2nd vote	47	48	49	50	3.3												
3rd vote	52	55	46	54	1.5												
3rd vote+	58	61	54	61	15.5												
1st vote/3rd vote	47	48	49	50	4.3												
1st vote/3rd vote+	51	54	46	53	4.0												
2nd vote/3rd vote	51	54	46	53	2.8												
2nd vote/3rd vote+	52	55	49	54	1.8												

as it is done in conventional elections, the optimal choice of a multifaceted representative is a rather difficult task. Therefore, a rational assistance is helpful.

To illustrate rigorously this approach using the 2025 *Wahl-O-Mat* questions, we would need the electors' individual policy profiles. Since they are unavailable, we do it most simplistically, assuming that the voters have expressed their multi-criteria evaluations through the votes they actually cast. Hence, we obtain the allocation of the Bundestag seats as it is — elected by the second votes. Column in Table 4 '2nd vote/A' displays the **A**bsolute percentages of the second votes received by the eligible parties, and Column '2nd vote/N' shows these percentages **N**ormalized after the votes for non-eligible parties have been excluded, i.e. the party quotas in the Bundestag.[4] For example, the CDU/CSU receive 29% of the second votes, and after excluding the non-eligible parties (that receive $< 5\%$ of the votes) and normalization of the percentages, the CDU Bundestag quota becomes equal to 33%. For the quotas defined according to the 2nd vote, the Bundestag popularity and universality indices computed in [Tangian 2025b], for unweighted and Google-weighted questions are shown in Row 'BUNDESTAG by…2nd vote' (47, 48, 49 and 50) and depicted at the beginning of the second row of Figure 2, where the bar length of each party is proportional to its Bundestag quota.

[4]Every A-column contains absolute percentage points (for parties' votes received or their indices of representativeness), and every N-column — normalized percentage points (reduced to the total of 100% — to be used as Bundestag quotas). The only exception is Column '1st vote (direct mandates)/A' that contains the (absolute) number of party's direct mandates. The footnote in page 1 explains why the total of this column is 276 instead of the number of constituencies 299.

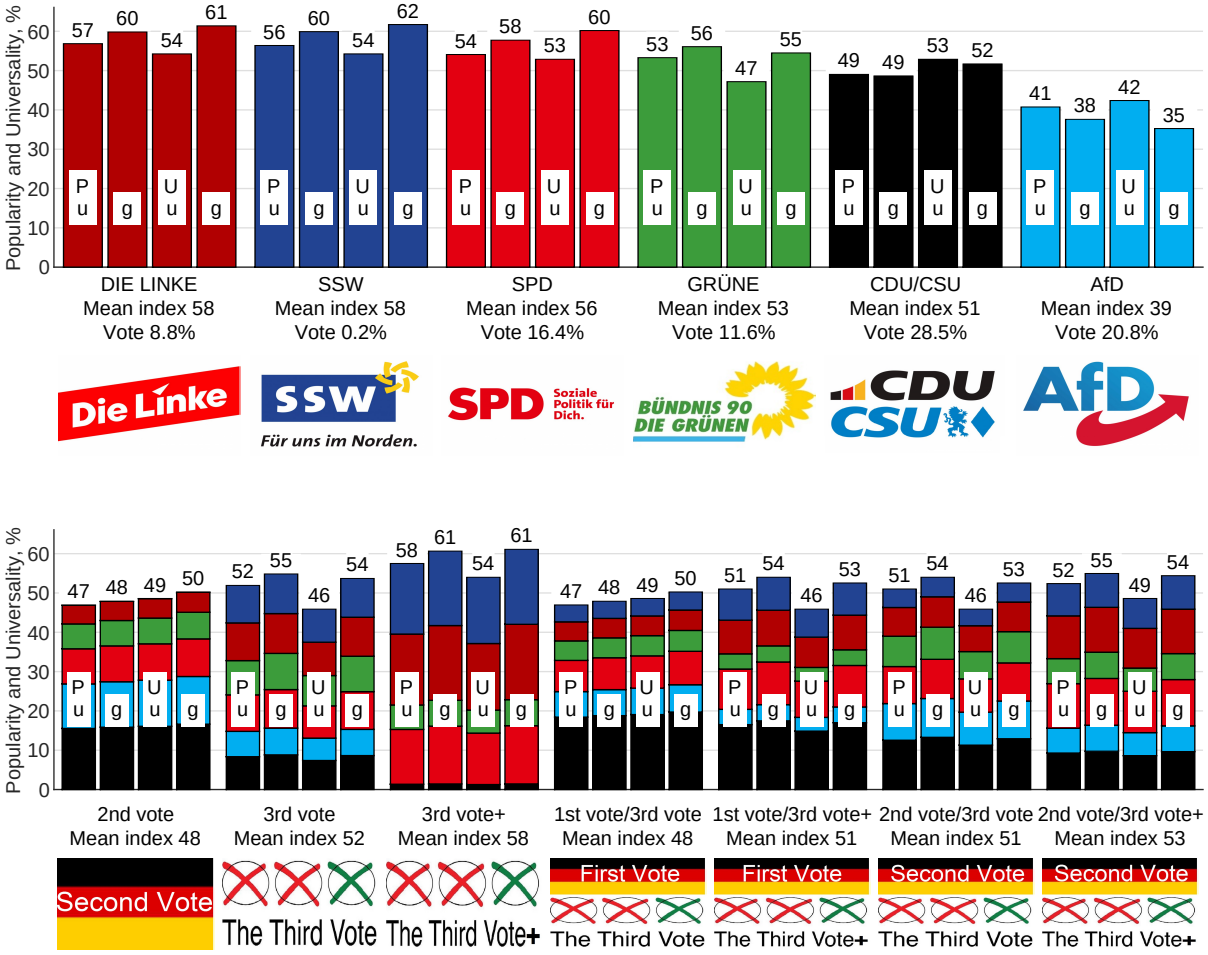

Figure 2: Visualization of Table 4. The 2025 indices of the Bundestag parties, the Bundestag and the hypothetical Bundestag as if elected using Third Vote methods: P—popularity, U—universality, u—for unweighted questions, and g—for Google-weighted questions.

3.3 Third Vote for public determination

The second architecture, shown in Table 3 as $\rightrightarrows\downarrow$, implements the philosophy of public determination. Under this architecture, the parties receive no votes even indirectly. Instead, the third votes[5] — answers to a questionnaire — are used to construct the policy profile of the electorate with the balance of public opinion on each issue. Then the eligible parties are indexed with respect to their policy representation ability — to reflect how well their policy profiles match with that of the electorate as a single body. For this purpose, we use the indices of popularity (the average percentage of the population represented) and/or universality (frequency in representing a majority). Then the parties' Bundestag quotas are defined in proportion to the parties' indices. Throughout the paper, this election method is called the *Third Vote* and denoted '3rd vote'.

Unlike the Advised Vote, the Third Vote does not lead to paradoxes. According to the philosophy of public determination, the Third Vote does not deal with numerous individual preferences but considers a single 'general will'. Obviously, a single preference ordering is not exposed to paradoxes.

We illustrate this approach using the 2025 *Wahl-O-Mat* questionnaire. As mentioned in the previous

[5]To avoid misunderstanding: the third votes (in small letters) are questionnaires in electoral ballots which are used either in the Advised Vote or the Third Vote election methods (capitalized).

paragraph, the policy profiles of individual electors are unavailable, but they are not really necessary for our purpose. It suffices to know the profile of the electorate as a whole — which corresponds to the result of horizontal aggregation in Table 3. In [Tangian 2025b], the policy profile of the electorate is revealed through the relevant public opinion polls. The degree of its matching with the parties' political profiles is measured by the mean of the parties' unweighted and Google-weighted popularity and universality indices. These indices are collected in Table 4 and illustrated in Figure 2. The **A**bsolute means of the four indices are given in Column '3rd vote $\frac{\mathsf{P}+\mathsf{U}}{4}$/A'. The means **N**ormalized (to use as the Bundestag's party quotas) are shown in Column '3rd vote/N', which suggests the hypothetical composition of the 2025 Bundestag as if elected by the Third Vote.

The popularity and universality of the Bundestag adjusted this way are given in Row 'BUNDESTAG by…3rd vote' and shown by the second block in the bottom section of Figure 2. As one can see, the *mean* representativeness of the 2025 Bundestag elected by the second votes is about 48%, meaning that it is 'non-representative rather than representative', whereas the indices of the Bundestag as if elected by the Third Vote are higher, with the mean of 52%, making it 'representative rather than non-representative'.

Row *Absolute Maximum* in Table 4 indicates the theoretical maximum of the indices — when for all questions, for which the balance of public opinion is known, only majorities in the society are represented; see [Tangian 2020, p. 284]. Since the index range is always symmetric about the central point of 50%, the index minimum follows from the maximum. For instance, the maximum popularity of 71% implies its minimum of 29%. The universality takes values between 0% and 100%, corresponding to the cases when no majority is represented and only majorities are represented, respectively.

The absolute maximum is a reference to judge how the representativeness potential is exhausted. Taking into account the popularity limits of 29–71%, the increase in popularity from 47–48% (for the Bundestag allocation according to the second vote) to 52–55% (for the Bundestag allocation according to the Third Vote) is quite significant — not just absolute 6–7% but 14–16% of the range.

4 Faction equalization effect and the Third Vote+

4.1 Equalization of quotas

Column 'Max quota ratio' in Table 4 the ratio between the largest and the smallest quota. For the Bundestag allocated according to the 2nd vote (Column '2nd vote/N'), the maximum quota ratio (neglecting the minority party SSW) is indicated in Row 'BUNDESTAG by 2nd vote':

$$\text{Max quota ratio under the 2nd vote} = \frac{\text{CDU/CSU quota}}{\text{LINKE quota}} = \frac{33\%}{10\%} = 3.3\ . \tag{1}$$

Comparing this ratio with the one of the Third Vote (LINKE/AfD=18%/12%=1.5), we see that the Third Vote equalizes the Bundestag factions.

The equalization effect is caused by the nature of the popularity and universality indices that determine the party quotas. Under conventional elections, political parties are voted on by disjoint groups of their adherents of very different sizes, implying the different sizes of the party factions, whereas under the Third Vote every party is also supported by the adherents of other parties who share the same position on the given issue. Instead of many groups of party adherents, on every issue there are only two quite numerous groups — Yes-group and No-group (not counting those who abstain), and the parties that share common views represent them jointly rather than exclusively. Correspondingly, the groups of party adherents join (differently on each issue), providing stronger support for weak parties. Such an extended account of the parties' policy representation ability, embodying the principle of proportional representation of public preferences in full, leads to a significant equalization of party quotas in the Bundestag. Therefore, the policy representation ability of big parties can be comparable with that of small parties. In addition, the parliament representativeness gains due to a more significant presence of

minor parties that contribute to a better representation of public opinion on numerous issues.[6]

By these reasons, the equalization effect should not be regarded harmful by itself only because we are not used to it. From the viewpoint of policy representation, it creates no problem at all and even can improve the parliament's performance.

In fact, there are two points, both beyond the policy representation we focus on here. The first is that the relations between parties and the electorate, understood in a broad sense, are not restricted exclusively to policy representation. There are ideological, religious, ethnic, cultural and intellectual reasons which cannot be ignored. Large parties are much closer to the electorate due to networking, traditions, voters' self-identification with parties, candidates' political experience and trustfulness, etc., which should be reflected in the party quota ratio.

The second point is coalition building. Since the equalization effect can reduce the factions of major parties, a ruling coalition may require three or even more member parties, complicating negotiations or making them impossible at all. If a parliament majority attains no consensus, the work of the parliament can hardly be efficient.

Thus, there are reasons to moderate the equalization effect. As we show below, it can be done either by slightly modifying the Third Vote or by combining it with the conventional vote by party name.

4.2 The Third Vote+

To tackle the equalization effect by modifying the Third Vote, we reconsider the indices that determine the party quotas in the Bundestag. From the indices of popularity and universality, we remove their 'non-representative' segment and retain only the segment above the critical threshold of 50%. In other words, we focus on the positive representativeness only, and, correspondingly, define the party quotas in proportion to the parties' indices of *positive* representativeness. This modification of the Third Vote method is called *The Third Vote+* and denoted '3rd vote+'.

Computationally, the mean of the popularity and universality indices is reduced by 50%: Column '3rd vote/A' in Table 4 is transformed into Column '3rd vote+/A'. Then the parties' negative values (meaning non-representativeness rather than representativeness) are nullified to exclude the parties from consideration, and then the index is normalized (the total reduced to 100%) — as shown in Column '3rd vote+/N' determining the Bundestag party quotas. Now the faction of AfD is nullified,[7] and the Bundestag's

[6]The former *Wahl-O-Mat*'s neglect of minor parties (before the 2009 Bundestag elections, the *Wahl-O-Mat* dealt with five major parties only) has been condemned by the Administrative Court of Cologne with the admonition that they must be given better chances:

> Just over a week before the European elections [on May 23–26, 2019], the Federal Agency for Civic Education (bpb) has shut down its Wahl-O-Mat. According to a ruling of the Administrative Court of Cologne, the internet offer, which is intended to provide orientation in elections, may not be continued for the time being. In its current form, it violates the constitutionally guaranteed right to equal opportunities, the judges ruled, thereby giving an application by the party Volt Germany. This party felt disadvantaged because the Wahl-O-Mat users can only compare up to eight parties. Smaller and as yet unknown parties would be disadvantaged in this way, the court said, making it clear that the Federal Agency for Civic Education was obliged to give each party the same opportunities in the election campaign.
>
> [dpa/Zeit 2019a, Court stops the Wahl-O-Mat].

For more information see [Ratzesberger 2019], and for the improved *Wahl-O-Mat* released a few days later see [dpa/Zeit 2019b].

[7]This purely theoretical count makes sense only if the elements of Column '3rd vote+/A' in Table 4 are all positive, i.e. all Bundestag parties are more representative than non-representative. Otherwise, the nullification of factions for the parties that owe direct mandates must be overcome; possible solutions are suggested in the following Section 5.

maximum party quota ratio is equal to

$$\text{Max quota ratio under the 3rd vote+} = \frac{\text{LINKE quota}}{\text{CDU/CSU quota}} = \frac{31\%}{2\%} = 15.5\ ,$$

which is even more contrasting than under the second vote. The popularity and universality indices of this version of Bundestag are displayed in Row '3rd vote+' of Table 4 and the second segment of Figure 2. As one can see, the mean index of the Bundestags allocated using the 3rd vote+ is greater than that of the Bundestag allocated using the 3rd vote by 6%. Regarding Row '*Absolute maximum*', the popularity increase from 52–55% to 58–61% implies 14% of the theoretical popularity range of 42% ($= 71 - 29$).

5 Combining the Third Vote with conventional election methods

5.1 Combining the Third Vote and Third Vote+ with direct mandates

To design an election method, one needs a reference index which is transformed into parliamentary quotas. In conventional election, this index is the percentage of votes received by the parties. To combine two election methods, it suffices to combine their reference indices.

We start with defining the first vote index, which is the percentage of direct mandates received by each party. For this purpose, the numbers of direct mandates in absolute figures in Column '1st vote/A' of Table 4 are normalized, that is, converted into percentages shown in Column '1st vote/N'.

To combine the Third Vote with the first vote, we take the mean of the first vote index in Column '1st vote/N' and the Third Vote index in Column '3rd vote/N', which is shown in Column '1st vote & 3rd vote'. As a mean of two normalized indices, this index is normalized and can be used as Bundestag quotas in %. The same procedure is valid for combining the first vote with the Third Vote+.

Using the mixed index in Column '1st vote & 3rd vote+/N' as the Bundestag quotas for eligible parties, the Bundestag's mean representativeness increases from 48% to 51%; see Row '1st vote/3rd vote+' of Table 4 and the fifth block in the bottom section of Figure 2. The popularity increases from 47–48 to 51–54 percentage points. Taking into account Row '*Absolute maximum*' of Table 4, this increase by 5–6% turns out to be a 12–14%-gain within the maximal theoretical range of 42% ($= 71 - 29\%$). At the same time,

$$\text{Max quota ratio under the 1st vote \& 3rd vote+} = \frac{\text{CDU/CSU quota}}{\text{AfD quota}} = \frac{32\%}{8\%} = 4.0\ ,$$

which is as common as the 3.3 obtained under the 2nd vote; see (1).

It should be noted that combining the 1st vote with the 3rd vote gives in our case no gain in the Bundestag representativeness but only an increase in the maximum quota ratio up to 4.3; see Table 4 and Figure 2.

5.2 Combining the Third Vote with vote by party name

The Third Vote can also be combined with the conventional vote by party name. Following the reasons from the previous subsection, we mix the indices of the 2nd vote and the 3rd vote or 3rd vote+. The mix can be done in any proportion, but we continue to use simple mean. For instance, Column '2nd vote & 3rd vote' of Table 4 shows the mean of Columns '2nd vote/N' and '3rd vote/N'. Using this mixed index to define the Bundestag quotas, we combine both voting methods. The same is valid for the mix in Column '2nd vote & 3rd vote+'.

The popularity and universality indices of the resulting Bundestag are displayed in Rows 'BUNDESTAG by 2nd vote/3rd vote' and 'BUNDESTAG by 2nd vote/3rd vote+' of Table 4 and in the bottom section of Figure 2. As one can see, the representativeness of such a Bundestag is much better than the representativeness of the Bundestag allocated in correspondence to the conventional second vote; cf. with Row 'BUNDESTAG by 2nd vote'.

The combinations of the 2nd vote with the Third Vote and Third Vote+ moderate the faction equalization effect of the latter. Indeed, using the data from Columns '2nd vote & 3rd vote' and '2nd vote & 3rd vote+' of Table 4, we obtain the figures at the end of Rows 'BUNDESTAG by 2nd vote/3rd vote' and 'BUNDESTAG by 2nd vote/3rd vote+':

$$\text{Max quota ratio for BUNDESTAG by 2nd vote \& 3rd vote} = \frac{\text{CDU/CSU quota}}{\text{SSW quota}} = \frac{25\%}{9\%} = 2.8\ ,$$

$$\text{Max quota ratio for BUNDESTAG by 2nd vote \& 3rd vote+} = \frac{\text{SPD quota}}{\text{AfD quota}} = \frac{22\%}{12\%} = 1.8\ .$$

6 Coalitions in the 2025 Bundestag under the Third Vote

Our discussion would be incomplete if limited to estimating only the Bundestag's representativeness before coalition building. Indeed, parliamentary factions usually unite into coalitions that finally determine the parliament decisions, so estimating the representativeness of possible coalitions is no less important than that of the Bundestag before the coalition building. Following [Tangian 2025b, Section 3], we analyze minimum eligible coalitions for seven compositions of the 2025 Bundestag as if elected by the seven methods listed in Table 4.

Table 5 and its visualization in Figure 3 have the same design as Table 5 and Figure 3 in [Tangian 2025b], respectively. The difference is that the coalitions here are given for seven alternative compositions of the Bundestag, with each election method being represented by one most unanimous (most compatible) minimum eligible coalition. In Figure 3, the election methods are distinguished using the flagstaffs: the red flagstaff (labeled '2' as in Table 5) distinguishes the coalition in the Bundestag elected using the 2nd vote, i.e., by party name. The green flagstaff (labeled '3') indicates the use of the 3rd vote, and the blue one (labeled '3+) — the use of the 3rd vote+. The mixed election methods are shown by dashed flagstaffs whose colors are associated with the election methods mixed and they are labeled respectively. For instance, the black/green flagstaff labeled '13' indicates the combination of the 1st vote and 3rd vote.

The representativeness of the most unanimous coalition depends, among other things, on the ratio of party quotas determined by the method with which the Bundestag has been elected. Therefore, the composition and the representativeness of the 'best' coalition depend on the election method. Ordering the non-mixed election methods (the first three in the table) by the coalition representativeness, we obtain the Third Vote+ (labeled '3+') at the top, then the Third Vote (labeled '3'), and then the second vote (labeled '2'). For the mixed methods, the order is as follows: the first vote and the Third Vote+ (labeled '13+') at the top, than the second vote and the Third Vote+ (labeled '23+') or the combination of the Third Vote with the first (labeled '13'), leaving behind the combination of the second and Third Vote (labeled '23').

The currently ruling coalition CDU/CSU+SPD is missing in Table 5 and Figure 3 because its unanimity is not the best for any election methods considered. The ruling coalition only appears in Table 6 and its visualization in Figure 4, where for each election method all minimum eligible coalitions with the unanimity down to 52% are displayed. For such a relaxed restriction, Coalition CDU/CSU+SPD appears three times: under the 2nd vote (labeled '2'), under the combination of the 1st and 3rd votes (labeled '13') and under the combination of the 1st and 3rd vote+ (labeled '13+'). In Figure 4, these coalitions are clustered close to coordinates (Popularity, Universality, Unanimity) $\approx$ (50.5, 52,5, 52.78) .

It should be emphasized that the real-world coalitions do not exclusively rely on the closeness of party positions, which in our model is measured by the unanimity index. There are also other important criteria, like ideological and personal compatibility, party image, size and influence, cooperation experiences, and some others. Therefore, it makes sense to consider a larger selection of minimum eligible coalitions, whose unanimity is not necessarily the best for each parliament composition, but still acceptable.

Table 5: Unweighted indices of the 2025 Bundestag most unanimous minimum eligible coalitions for seven election methods and faction weight factor $f = 0.5$

Election method Coalition	Faction/coalition weights	Unanimity	Popularity		Universality	
			Expectation	Standard deviation	Expectation	Standard deviation
	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank
2 (2nd vote)						
CDU/CSU+AfD	33+24=57/3	75.00/4	46.39/7	±2.03/7	52.45/3	±4.15/5
3 (3rd vote)						
SPD+GRÜNE+SSW	18+17+18=53/5	81.08/2	53.58/5	±1.52/2	48.83/6	±3.54/2
3+ (3rd vote+)						
LINKE+SSW	31+31=63/1	89.19/1	57.65/1	±0.97/1	56.76/1	±2.70/1
13 (1st vote/3rd vote)						
CDU/CSU+AfD	39+14=53/6	75.00/4	46.94/6	±1.98/6	51.78/4	±4.05/4
13+ (1st vote/3rd vote+)						
SPD+LINKE+SSW	20+17+16=52/7	72.97/5	55.82/2	±1.72/3	53.68/2	±4.24/6
23 (2nd vote/3rd vote)						
SPD+GRÜNE+LINKE+SSW	18+15+14+9=57/2	70.27/6	54.72/4	±1.91/5	46.43/7	±4.41/7
23+ (2nd vote/3rd vote+)						
SPD+GRÜNE+LINKE	22+12+21=55/4	75.68/3	55.06/3	±1.79/4	51.64/5	±4.03/4

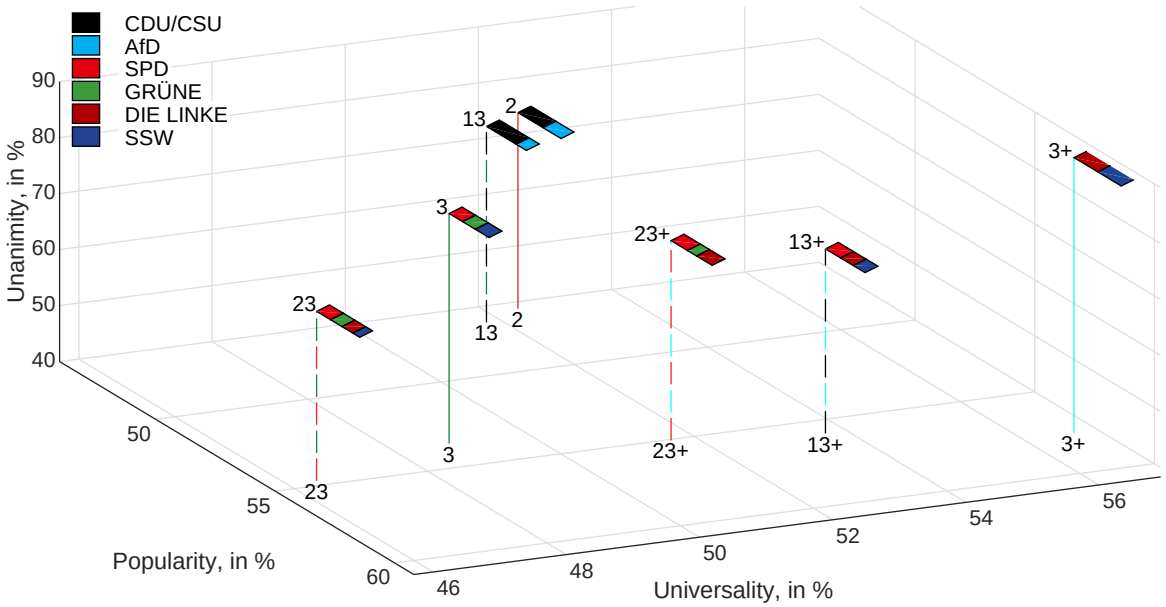

Figure 3: Visualization of Table 5. Election method by flagstaff: red — 2 (2nd vote), green — 3 (3rd vote), blue — 3+ (3rd vote+), black/green — 13 (mix of 1st and 3rd votes), black/blue — 13+ (mix of 1st vote and 3rd vote+), red/green — 23 (mix of 2nd and 3rd votes), red/blue — 23+ (mix of 2nd vote and 3rd vote+).

Table 6: Unweighted indices of the 2025 Bundestag minimum eligible coalitions with unanimity $> 52\%$ for seven election methods and faction weight factor $f = 0.5$

Election method Coalition	Faction/coalition weights	Unanimity	Popularity		Universality	
			Expectation	Standard deviation	Expectation	Standard deviation
	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank
2 (2nd vote)						
CDU/CSU+AfD	33+24=57/4	75.00/7	46.39/16	±2.03/13	52.45/11	±4.15/9
CDU/CSU+SPD	33+19=52/16	52.78/10	50.49/13	±2.88/15	52.59/9	±5.67/15
3 (3rd vote)						
SPD+GRÜNE+SSW	18+17+18=53/12	81.08/4	53.58/11	±1.52/3	48.83/15	±3.54/4
GRÜNE+LINKE+SSW	17+18+18=54/10	78.38/5	55.37/6	±1.76/8	56.18/3	±3.78/5
SPD+GRÜNE+LINKE	18+17+18=53/11	75.68/6	54.96/8	±1.79/10	49.25/14	±4.03/7
SPD+LINKE+SSW	18+18+18=55/8	72.97/8	55.91/2	±1.72/5	59.00/1	±4.24/10
3+ (3rd vote+)						
LINKE+SSW	31+31=63/1	89.19/1	57.65/1	±0.97/1	56.76/2	±2.70/1
SPD+LINKE	24+31=56/6	83.78/2	55.70/5	±1.37/2	54.05/4	±3.30/2
SPD+SSW	24+31=56/7	83.33/3	54.83/9	±1.57/4	52.78/7	±3.40/3
13 (1st vote/3rd vote)						
CDU/CSU+AfD	39+14=53/13	75.00/7	46.94/15	±1.98/12	51.78/12	±4.05/8
CDU/CSU+SPD	39+17=56/5	52.78/10	50.37/14	±2.85/14	52.50/10	±5.61/14
13+ (1st vote/3rd vote+)						
SPD+LINKE+SSW	20+17+16=52/14	72.97/8	55.82/4	±1.72/7	53.68/5	±4.24/12
CDU/CSU+SPD	32+20=52/15	52.78/10	50.52/12	±2.88/16	52.62/8	±5.69/16
23 (2nd vote/3rd vote)						
SPD+GRÜNE+LINKE+SSW	18+15+14+9=57/3	70.27/9	54.72/10	±1.91/11	46.43/16	±4.41/13
23+ (2nd vote/3rd vote+)						
SPD+GRÜNE+LINKE	22+12+21=55/9	75.68/6	55.06/7	±1.79/9	51.64/13	±4.03/6
SPD+LINKE+SSW	22+21+16=58/2	72.97/8	55.83/3	±1.72/6	53.67/6	±4.24/11

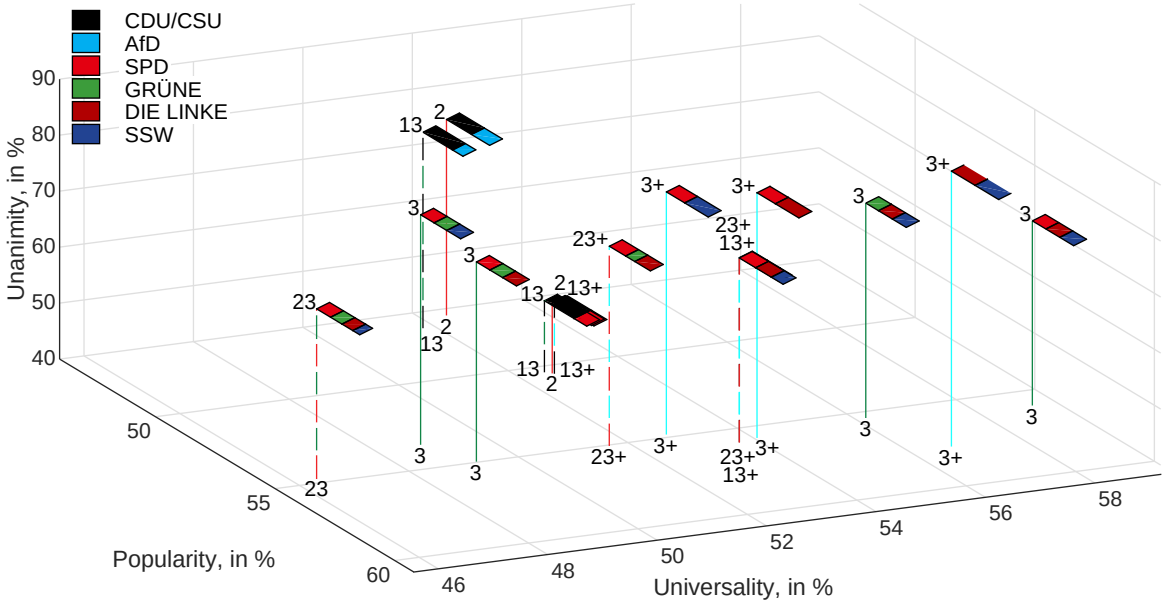

Figure 4: Visualization of Table 6. Election method by flagstaff: red — 2 (2nd vote), green — 3 (3rd vote), blue — 3+ (3rd vote+), black/green — 13 (mix of 1st and 3rd votes), black/blue — 13+ (mix of 1st vote and 3rd vote+), red/green — 23 (mix of 2nd and 3rd votes), red/blue — 23+ (mix of 2nd vote and 3rd vote+).

7 Summary: Enhancing policy representation

The two concepts of political representation introduced at the end of the 18th century, the descriptive and agent ones, were thoughtfully embodied in historical election systems. This is not the case of the concept of policy representation which was coined during the last third of the 20th century.

To be implemented, this concept needs a dedicated election method that could make representative democracy 'more representative'. To meet this objective, we discuss the Third Vote election method, which explicitly takes into account the voters' policy preferences. To be specific, the Third Vote is applied to the data of the 2025 German Bundestag elections. The hypothetical re-distribution of the Bundestag seats among eligible parties results in a significant gain in the Bundestag's policy representation ability. The same effect was observed in experiments during the 2016–2018 elections of the Student parliament of the Karlsruhe Institute of Technology [Tangian 2017c, Tangian 2020, Chapters 15–17]; see also [World Forum for Democracy 2016, 2019].

To conclude, this paper continues the discussion of the 2023/24 electoral reform in [Tangian 2025a]. If the paper cited is about improving the Bundestag apportionment accuracy and enhancing the descriptive concept in the German electoral system, then this article suggests modifications to the conventional election procedure which could enhance the policy representation concept.

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