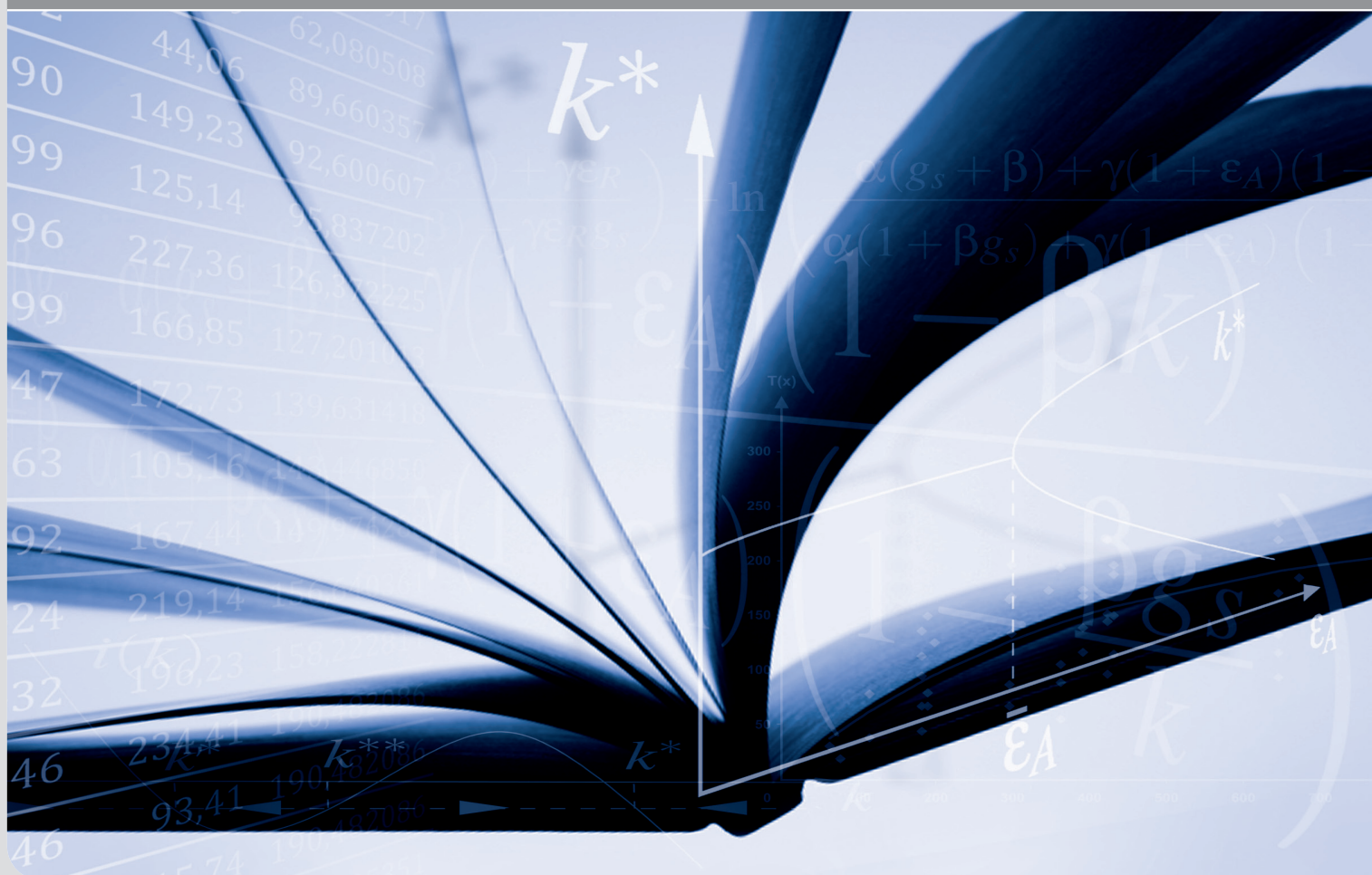


Analysis of the 2025 Bundestag elections. Part 4 of 4: Changes in the German political spectrum

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Analysis of the 2025 Bundestag Elections 4/4. Changes in the German Political Spectrum¹

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Abstract

This is the last out of four papers on the 2025 German federal elections continuing our analysis of the 2009, 2013, 2017 and 2021 Bundestag elections. First, we apply the model from [Tangian 2022b] to construct the 2025 German political spectrum understood as a contiguous party ordering, i.e., such that the neighboring parties have close policy profiles. For this purpose, we consider the parties that took part in the 2025 federal elections, define their policy profiles as 38-dimensional vectors of their Yes/No answers to 38 policy questions from the German voting advice application *Wahl-O-Mat* ('Support for Ukraine'?—Yes/No, 'General speed limit on motorways'?—Yes/No, etc.), and contiguously order them by means of Principal Component Analysis. The circular party ordering obtained is cut, resulting in a horseshoe-shaped left-right ideological axis with the far-left and far-right ends approaching each other. Among other things, the one-dimensionality of the political spectrum looks as a precondition for the voters' single-peaked preferences that guarantee the election consistency.

Second, using similar data from the 2009, 2013, 2017 and 2021 German federal elections, we construct political spectra for these years as well and trace the changes. Since the set of contesting parties varies from one election to another, and the *Wahl-O-Mat* questions vary as well, we only dispose five party orderings with a relatively small core of 13 parties that participated in all five elections. To locate the five spectra on a common scale, we consider 60 parties that have ever participated in elections and order them basing on five spectra on subsets of 24, 29, 31, 37 and 28 parties, respectively. This is done in terms of collective choice: find a group preference on 60 alternatives given five individual preferences on five incomplete alternative subsets; so we adapt the Condorcet and Borda approaches. Then the five political spectra are stretched onto this unified party ordering by constrained least squares, adjusting the distances between the parties in each spectrum.

All of these enable to adequately visualize party reshuffles in the political space. In particular, we see that, among the major German parties, the SPD fluctuates by far the most between left and right. This political inconsistency can deter voters, especially floating voters without a firm self-identification with a particular party, and may explain the SPD's failure in the 2025 elections, when the party received the historical minimum of 16.4% of the votes, having lost 9.3 percent points compared with the 2021 elections.

Keywords: Political spectrum; contiguous party ordering; left-right ideological axis; single-peaked preferences; principal component analysis, group choice with incomplete individual preferences.

JEL Classification: D71

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The existence of an informed and interested electorate, that may be swayed one way or the other, creates an incentive for politicians to put policy proposals directly to the public. The consent of a majority on policy measures can be built up within the electorate itself. Discussion of specific issues is no longer confined to Parliament (as in parliamentarianism), or to consultation committees between parties (as in party democracy); it takes place within the public. Thus, the form of representative government that is emerging today is characterized by a new protagonist of public discussion, the floating voter, and a new forum, the communication media.

Bernard Manin (1997) *Principles of Representative Government*, p. 231.

1 Introduction

This is the last out four papers on the 2025 German Bundestag elections continuing our analysis of the elections in 2009, 2013, 2017 and 2021 [Tangian 2014, 2020, 2022a–d]. The structure of the first part of the paper follows [Tangian 2022b], from which we quote for the reader’s convenience without special reference.

The location of a party in ‘political space’ is the central question of most theories of political competition. Since [Smithies 1941, Downs 1957] — for comments see [Van Houweling and Sniderman 2005] — this question has been extensively elaborated. Although the objectivity of political space is sometimes called into question [Benoit and Laver 2012, Otjes and Louwerse 2014], a number of particular directions have been developed.

For instance, [Hinich and Munger 1994], and somewhat later [Poole 2005, Poole and Rosenthal 2007, Carroll et al. 2013], created a theory of ideological space which was applied to ‘dimensionalize’ the U.S. Congress’ ideology, having overcome the paradox of low-dimensionality using unidimensional scaling with the least squares metric. The spatial theory of elections by [Enelow and Hinich 1984, Enelow 1994, Enelow and Hinich 1990, Hinich and Munger 1997] was developed further by [Saari 1994, Saari 1995, Kriesi et al. 2006, Kriesi 2008, Armstrong II et al. 2014, Wheatley et al. 2014, Wheatley 2015]. Several authors attempt to order parties along the left-right ideological axis, although there is no general consensus on such an arrangement [Luther 2012, Mair 2007, Müller-Rommel and Bétoa 2016, Neundorf 2009, Neundorf 2011].

Ordering parties linearly is aimed not only at the visualization of political space. ‘Simple’ political spectra contribute to dispelling doubts in the consistency of elections in light of theories ranging from Condorcet’s paradox regarding cyclic majorities to Arrow’s assertion on the ‘impossibility’ of rational collective decision-making; such doubts are considered an obstacle [Nurmi 1999, Gehrlein and Lepelley 2011]. To avoid logical inconsistencies in the collective preference, numerous scholars introduce domain restrictions, that is, conditions that constrain the choice of individual preferences; see the dedicated monograph [Gaertner 2001] and related sections in [Arrow et al. 2002/2011]. The best known domain restriction, the so-called single-peaked preferences, is due to [Black 1948, Black 1958], whose discovery marks the origin of the public choice theory. It is assumed that the given alternatives can be ordered *in a line* along which the preference of each voter increases until a certain voter-determined maximum, whereupon it drops off. Black proves that such single-peaked preferences result in no cyclic majorities; see also [Ballester and Haeringer 2011, Moulin 1988, Puppe 2018]. This framework has been generalized to multidimensional single-peakedness [Barberà 2011, Sui et al. 2013]. The idea of avoiding cyclic majorities by some linear alignment, e.g. of voters or whatever else, is implemented in the notions of single-crossing preferences, 1-Euclidian preferences, top-monotonicity, etc.; see [Grandmont 1978, Barberà et al. 1993, Saporiti and Tohmé 2006, Saporiti 2009, Barberà 2011, Barberà and Moreno 2011, Skowron et al. 2013, Elkind et al. 2014].

Being a theoretical assumption, the single-peakedness in its pure form is seldom observed in real-world situations [Conitzer 2009, Escoffier et al. 2008]. However, Condorcet cycles, which signal inconsistency in elections, occur in practice much less frequently than the theory predicts, making some scholars be-

lieve in some ‘natural’ regulating mechanisms [Grofman and Feld 1988, Young 1988, Gehrlein 2002]. If we compare the Condorcet count, which leads to cyclic majorities, with the Borda count, where cycles never emerge, we see that the results often coincide, implying that the Condorcet count causes no cyclic majorities either [Tangian 2020, Section 4.9]. This phenomenon was recognized by Condorcet himself:

It is even highly probable that this [Borda’s] method would only rarely lead to an error about the true plurality decision.

[Condorcet 1785, *Essai*...; cited from [Condorcet 1994], p. 138]

The election consistency observed allows theorists to assume in elections a certain one-dimensionality with single-peakedness, even if distorted. In other words, single-peaked preferences are not considered to be exclusive but rather a core of some larger domain of individual preferences that still do not produce cyclic majorities. This is sometimes expressed in terms of probabilities, suggesting that opposite random deviations from single-peakedness cancel each other out and thereby do not affect the transitivity of the majority preference [Regenwetter et al. 2006]. Even without considering probabilities, minor violations of single-peakedness, especially in large settings, are unlikely to change the overall picture, justifying the notion of approximate single-peakedness [Bredereck et al. 2013, Sui et al. 2013]. There is also empirical evidence that an approximate single-peakedness arises in meaningful voting situations, as in deliberative polls [List et al. 2013]. The same is empirically revealed in elections, where voters frequently refer to the left-right political scale [Züll and Scholz 2015], which creates preconditions for approximate single-peaked preferences.²

The reference to the left-right political axis is justified by the fact that, until recently, class opposition has been regarded as the major political driver. Correspondingly, this axis has been used to delineate political agents [Lipset 1960, Rous and Lee 1978, Mahoney et al. 1984, Bobbio 1996, Gauchet 1996, Ware 1996, Wilson 2004, Ruypers et al. 2005, Knapp and Wright 2006, Blattberg 2009]. It has also been used to locate the electors themselves, forming a precondition for single-crossing preferences, which constitute another important domain restriction to providing election consistency [Mirrlees 1971, Roberts 1977, Saporiti and Tohmé 2006, Saporiti 2009, Cornaz et al. 2013, Skowron et al. 2013, Elkind et al. 2014].

In recent years, the explanation of election consistency due to the left-right axis — and even the very meaning of ‘left’ and ‘right’ — have been called into question. Discussing the radical changes in the world order at the end of the 20th century, some political scientists began promoting the viewpoint that the traditional left-right alignment of parties is becoming outdated [Giddens 1994, Manin 1997, Mitchell 2007, Sulakshin 2010, Voda 2014]. It is argued that, after the fall of the Soviet Union and Eastern Block, the class opposition movement lost the impetus of its inspiration to a systemic alternative, which swayed public attention away from left-right political confrontations toward less ideological and more pragmatic matters. It should be noted that marginalization of the left-right opposition would deprive the European welfare state concept of its defense by social democrats and the left, paving the way for its replacement by the Anglo-Saxon model and Americanization of Europe.

²Historically, politicians were first called ‘left’ and ‘right’ during the French Revolution of 1789. In the National Assembly, which was replaced in 1791 by the Legislative Assembly and succeeded the National Convention in 1792, the supporters of the king were seated to the president’s right (the party of order) and supporters of the revolution to his left (the party of movement). In the 19th century, these terms were associated with the class divisions of the society. Following [Marx 1867] and [Weber 1921], economists and sociologists consider classes as social groups with common interests determined by income, property, education, social status, and their relation to the means of production. Their competing interests result in the class opposition headed by the ‘left’ or ‘right’ political parties that emerged after the Industrial Revolution. The left (anarchists, anti-capitalists, anti-imperialists, autonomists, communists, democratic-socialists, feminists, greens, left-libertarians, progressives, secularists, socialists, social-democrats and social-liberals) stand for egalitarianism, solidarity with income redistribution, and governmental intervention in the economy [Gosse 2005]. The right (capitalists, conservatives, fascists, monarchists, nationalists, neoconservatives, neoliberals, reactionaries, right-libertarians, social-authoritarians, theocrats and traditionalists) defend private property, free entrepreneurship and equal opportunities [Carlisle 2005, Knapp and Wright 2006, McLean and McMillan 2009].

From all of these, it is concluded that the political spectrum is becoming multidimensional, replacing the former left-right ideological alignment. This view is reflected in the MANIFESTO project, with its over 400-dimensional tabular representation of party programs from more than 50 countries covering all free democratic elections since World War II [Budge et al. 2001, Klingemann et al. 2006, Budge and McDonald 2007, Linhart and Shikano 2007, Volkens et al. 2013, WZB 2019]. Similarly, the VAAs (voting advice applications) implemented in about 20 countries and at the level of the EU assume multiple cleavages, i.e., multidimensional political spectra [Kieskompas 2006, You vote EU 2019, Gemenis 2013, Garzia and Marschall 2014, Vote Match Europe 2019]. Furthermore, VAAs have already been used to assess the dimensionality of a political space [Wagner and Ruusuvirta 2012, Wheatley 2012, Mendez and Wheatley 2014, Otjes and Louwerse 2014, Wheatley et al. 2014, Wheatley 2015].

In this paper, we empirically construct the 2025 political spectrum of Germany (and make the same for 2009, 2013, 2017 and 2021 — the years for which the data required are available). For this purpose, we test the thesis of multiplicity of equally significant political dimensions using the data from the *Wahl-O-Mat* — the voting advice application of the German Federal Agency for Civic Education [Bundeszentrale für politische Bildung 2025].

Before each of the federal elections of the years mentioned, the *Wahl-O-Mat* has formulated 38 dichotomous questions on the most topical policy issues (A general speed limit should apply on all motorways?—Yes/No, Germany should increase its defense spending?—Yes/No, etc.) and addressed them to the contesting parties. The parties Yes/No-answers in Table 1, constitute their ‘policy profiles’ used to define the proximity between the parties and locate them in the policy space.

The statement in question, that the party space is essentially multidimensional, would imply that the party vectors are scattered throughout this space more or less homogeneously, resulting in a ball-shaped cloud of ‘observations’. However, Principal Component Analysis (PCA), when applied to the parties’ proximity (correlation) matrix, reveals that the parties constitute a flat ellipsoid whose two longest diameters cover over 93% of the total variance.³ Reducing the model to these two dimensions, a one-dimensional contiguous party ordering is found that resembles the left-right axis rolled into a circumference. Such a curved one-dimensional axis (the one-dimensionality is understood in the topological sense) differs from the straight left-right ideological axis tested by the political scientists cited. It reflects the fact that the far-left and far-right ends, instead of being opposite, approach each other, although they do not touch, so that the political spectrum is Ω -shaped, i.e., looks like a horseshoe. Indeed, both extreme left and extreme right parties are populist, though with different backgrounds: they appeal primarily to the same lower classes, and they exhibit similarities in their positions on many policy issues supported by large fractions of the population.

This empirical finding meets the horseshoe theory attributed to [Faye 1996], which points to the closeness of the far-left and the far-right. Similar ideas, being inspired by works of Lipset (1922–2006) and Bell (1919–2011), are promoted by the US Pluralist School [Politicalresearch.org 2021]:

It may be more useful to think of the Left and the Right as two components of populism, with elitism residing in the Center. The political spectrum may be linear, but it is not a straight line. It is shaped like a horseshoe.
[Taylor 2006, *Where Did the Party Go?*, p. 118]

It should be emphasized that the left-right axis does not arise out of normative assumptions but is found ‘objectively’ — from the party positions on issues that are not directly linked to any ideology. This empirical evidence contradicts the assertion that the left-right axis is outdated. At the same time, the circularity of the political spectrum explains why linear empirical models fail to recognize its one-dimensionality [Sulakshin 2010, Voda 2014]: a circumference, being itself one-dimensional, cannot be placed in a one-dimensional Euclidian space; to be accommodated it needs a Euclidian space with at least

³Another methodology to analyze VAA spatial maps — dynamic scale validation (DSV) — is applied by [Germann et al. 2015, Germann and Mendez 2016].

two line axes. Here, we come to multiple-dimensional political spectra introduced by [Ferguson 1941, Eysenck 1955, Rokeach 1973]; for a review of later developments see [Mitchell 2007, Heywood 2017]. We locate the parties in a kind of Nolan’s 2D diagram [Heywood 2017] but the configuration they produce can be regarded as a 1D construct. Thus, our finding bridges two types of spatial political models [Gill and Hangartner 2010, Sect. 8]: (1) directional models of successive policy shifts with circular representations and angular measures [Grofman 1985, Linhart and Shikano 2007, Matthews 1979, Rabinowitz and Macdonald 1989, Schofield 1985], and (2) proximity models that describe the distance between political agents in the Euclidian space with line axes.

Comparing the German political spectra, we see that there is a certain development. Firstly, the political spectra are becoming progressively ‘more flat’: in 2009, 2013, 2017, 2021 and 2025, the two largest diameters of the ellipsoids of the party vectors, as revealed by PCA, cover respectively 74%, 80.6%, 84.3%, 87.3% and 93% of the total variance. In other words, the contiguous party orderings derived from similar data — party answers to 38 *Wahl-O-Mat* questions — are becoming more and more accurate.

The second observation is the relocation of certain parties along the left-right axis. It looks that some parties step back from their established ideological images and move in the political space attempting to find a more demanded niche to the end of gaining more votes — in line with the market-like theory of democracy by [Schumpeter 1942]. It seems that the German society, having previously been most attentive to the parties’ left-right orientation, is now also becoming sensitive to some other criteria. Since the candidates offer something to maximize public demand, politics acquires market characteristics, as reflected in numerous studies of the ‘electoral market’ [Manin 1997, p. 224]. Drawing analogy to the concepts of demand economy and supply economy, which differ in whether they respond to demand or to supply, we can speak of a shift from ‘demand politics’ to ‘supply politics’.⁴ This means that the freedom of public opinion is overridden by the preemptive political offer. Our days, trial by discussion (the term introduced by B. Manin) occurs not only at parliament or party meetings but also in the media. Politicians argue directly with interest groups, attempting to win ‘floating voters’ who are not adherents of any party and whose number is on the rise because of flexible candidate positions.

Compared with the earlier political spectra, the left parties in the 2025 spectrum are still well clustered. The right parties are clustered as well but less densely. The novel factor is the emergence of an intermediate cluster consisting of small parties with little ideology but populist claims between the far-left and the far-right ones. The effect is bridging the far-left and far-right ends of the horse-shoe-shaped spectrum and splitting it at the opposite side — between the libertarian left and the libertarian right, as if turning the horse-shoe upside down. Regardless of the splitting point (which we also discuss due course), the spectrum remains one-dimensional still providing precondition for consistent elections. (Single-peaked preferences even on a circular axis mostly lead to a transitive majority preference [Peters and Lackner 2020].)

In Section 2, ‘Political spectrum as a contiguous party ordering’, the data and methods for constructing political spectra are introduced. In particular, it is shown that the party ordering by votes received in an election is not contiguous, i.e., does not reflect the parties’ proximity in the political space.

In Section 3, ‘Political spectra obtained using dimensionality reduction’, the party space is reduced to one and two PCA principal components, respectively. While both resulting party orderings yield left-right alignments, the one obtained using two principal components is much more accurate.

In Section 4, ‘Left-right axis as a solution to the Traveling Salesman problem’, a contiguous party ordering is understood as the shortest itinerary when the parties are regarded as destinations and the inverted correlations $(1 - \rho)$ between their profiles are considered pseudo-distances.

In Section 5, ‘Solutions using weighted squares criteria’, the parties are ordered by minimizing the weighted squared distances between proximate parties or, alternatively, by maximizing the weighted

⁴ According to the founder of the economic approach to democracy, Joseph Schumpeter (1883–1950), in politics, demand is inseparable from supply. For instance, the common people have no independent opinion on the issues much beyond their own living circumstances, like in international affairs, where their ‘demand’ is conditioned by the governmental ‘supply’; see [Schumpeter 1942, p. 258].

squared distances between opposite parties.

In Section 6, ‘Choosing the party ordering to be regarded as political spectrum’, mathematical arguments and informal reasons are summarized to consider the party ordering obtained using the 2D PCA model as the German political spectrum.

Regardless of the party ranks, it becomes clear that the electoral success of a party depends neither on its policy representation capability nor on its left-right orientation.

In Section 7, ‘Evolution of the German political spectrum’, we use the 2D PCA model to additionally construct the 2009, 2013, 2017 and 2021 German political spectra. To facilitate comparisons, we bring the five German spectra to a ‘common denominator’ — a united political spectrum to be used as a reference axis. For this purpose, we develop a model to order a set of alternatives basing on orderings of its subsets, and apply it to construct a united spectrum basing on the five political spectra found. Locating the five spectra in a space with the united spectrum as its reference axis, we trace the party dynamics, repositioning and grouping. In particular, the model reveals significant fluctuations of the SPD between left and right, and this political inconsistency could be the cause of the SPD’s electoral failure in 2025.

In Section 8, ‘Conclusions’, the main findings are recapitulated and put into context.

Section 9, ‘Appendix 1: The 2025 Wahl-O-Mat questions’, lists the 2025 *Wahl-O-Mat* questions.

Section 10, ‘Appendix 2: The parties which participated at least in one of the 2009–2025 German federal elections’, contains lexicographically ordered descriptions of 60 German parties considered in this paper.

2 Political spectrum as a contiguous party ordering

Our goal — constructing the political spectrum of Germany — is to arrange the German parties in a contiguous way, i.e., so that the neighboring parties have close policy profiles defined as the 38-dimensional vectors of the party Yes/No answers to 38 policy questions shown in columns of Table 1. In Sections 2–5, we construct eight alternative party orderings for the 2025 data, using eight different models, and select the model with which we construct political spectra for other years.

Now we focus on technical details, and political implications are discussed at the end of the paper.

2.1 Proximity of party profiles

Since all five German political spectra are constructed absolutely in the same way and are based on the same data sources, we explain their construction using the example of the 2025 spectrum.

In 2025, 29 parties took part in the federal elections. One minor party, ‘Verjüngungsforschung’ (Rejuvenation Research), having dealt exclusively with gerontology, did not answer to the *Wahl-O-Mat* questions, so it is excluded from consideration. The conservative union of the CDU and the CSU answered the *Wahl-O-Mat* questions jointly, and it is regarded as one party. Hence, we have 27 parties whose answers to 38 *Wahl-O-Mat* questions are shown in Table 1; for the formulation of the questions, both in English and German, see Section 9, and for more detailed information about the questions see [Tangian 2025b, Section 5].

Our goal — constructing the German political spectrum — is to spatially arrange the German parties in a contiguous way, i.e., so that the neighboring parties have close policy profiles. The policy profiles — columns of Table 1 with ‘–’, ‘?’ and ‘+’ coded by -1 , 0 and $+1$, respectively⁵ — are considered

⁵A missing answer does not necessarily mean neutrality, which can indeed be coded with 0. For instance, there is evidence reported on the Québec and Scotland independence referenda [Durand 2015]: 2/3 of those who had abstained from a judgment in a pre-referendum poll ultimately voted ‘No’ (for the status quo) at the referendum, resulting in divergence between poll outcomes (where missing answers were interpreted as indifference) and referenda outcomes (with disclosed positions). Replacing missing values can be justified or called into question by the MCAR test (missing completely at random) [Little 1988, Little and Rubin 2002], which is however beyond the

Table 1: German party positions on the 2025 *Wahl-O-Mat* questions: +[1]—Yes, −[1]—No, ?—Neutral or missing

Question	Party positions																			
	CDU/CSU	AfD	SPD	GRÜNE	DIE LINKE	BSW	FDP	FREIE WÄHLER	Tierschutzpartei	Volt	Die PARTEI	dieBasis	Bündnis Deutschland	SSW	ÖDP	Team Todenhöfer	PdF	MLPD	PdH	PIRATEN
1 Support for Ukraine	+	+	+	+	−	−	+	+	+	+	?	−	?	+	+	−	+	−	+	?
2 Renewable energies	+	−	+	+	+	−	−	+	+	+	+	−	−	+	+	+	+	+	+	?
3 Cancellation of the citizen's allowance	+	+	+	−	−	+	+	+	−	−	−	+	+	−	−	+	+	−	−	?
4 Speed limit on motorways	−	−	+	+	+	−	−	+	+	+	?	−	+	+	?	+	+	−	?	−
5 Rejection of asylum seekers	+	+	−	−	−	+	+	+	−	−	−	+	+	−	?	−	+	−	−	+
6 Limitation of rental prices	+	−	+	+	+	+	−	−	+	+	+	+	−	+	+	+	+	−	+	−
7 Automated facial recognition	+	+	−	−	−	?	−	−	−	−	−	−	?	−	−	−	−	−	−	−
8 Energy-intensive companies	+	−	+	+	+	−	−	+	−	−	−	?	−	+	−	−	+	−	?	+
9 Pension after 40 years of contributions	−	−	−	−	+	+	−	+	+	+	+	+	−	+	?	+	+	?	+	+
10 Basic Law	+	+	+	?	−	?	?	+	−	−	−	+	+	−	+	+	−	−	−	+
11 Recruitment of skilled workers	+	?	+	+	?	?	+	+	+	+	+	−	+	+	?	+	+	−	+	−
12 Use of nuclear energy	+	+	−	−	−	−	+	−	−	−	−	+	+	−	−	−	−	+	−	+
13 Raising the top tax rate	−	−	+	+	+	+	−	−	+	+	+	?	−	+	+	+	+	+	+	−
14 Competencies in school policy	−	−	+	+	+	+	+	+	+	+	+	−	−	−	−	+	+	?	+	−
15 Arms exports to Israel	+	?	+	+	−	−	+	+	?	?	?	−	+	?	−	−	?	−	?	−
16 Health insurance companies	−	−	+	+	+	+	−	+	+	+	+	+	−	+	+	+	+	+	+	−
17 Abolition of the woman's quota	−	+	−	−	−	?	+	−	−	−	−	+	+	−	+	?	?	+	+	+
18 Organic farming	−	−	+	+	+	?	−	?	+	+	+	+	+	+	+	+	+	+	−	+
19 Projects against right-wing extremism	+	−	+	+	+	+	+	+	+	+	+	−	−	+	+	+	+	+	+	−
20 Monitoring of suppliers	−	−	+	+	+	?	−	−	+	+	+	?	−	+	+	+	+	+	+	−
21 Parent-dependent BAföG	+	+	+	+	−	+	−	−	−	−	−	+	+	−	−	−	−	−	−	+
22 Public debt brake	+	+	−	−	−	?	+	+	−	−	−	+	+	−	−	+	−	?	−	−
23 Work permit for asylum seekers	−	−	+	+	+	−	?	+	+	+	+	+	+	+	+	+	+	+	+	+
24 Abandoning climate targets	−	+	−	−	−	−	−	−	−	−	−	+	+	−	−	−	−	−	+	+
25 35-hour week	−	−	?	−	+	?	−	−	+	+	+	?	−	?	+	−	−	+	?	−
26 Abortion after counseling	+	+	−	−	−	−	?	+	−	−	−	+	+	−	+	+	−	−	−	+
27 National currency	−	+	−	−	−	−	−	−	−	−	−	?	−	−	−	−	−	?	−	−
28 Rail before road	−	−	+	+	+	+	−	?	+	+	+	+	−	+	+	+	+	+	−	+
29 Voluntary work	−	−	−	+	?	?	−	+	+	+	+	+	−	+	+	+	+	+	?	−
30 Allocation of property tax	+	?	−	−	−	−	+	+	−	−	−	?	+	−	+	−	+	−	?	−
31 Restriction of the right to strike	?	+	−	−	−	−	+	+	−	−	−	−	+	−	−	−	−	−	−	?
32 Referendums	−	+	?	−	+	+	−	+	+	−	+	+	+	−	+	+	+	+	−	+
33 Criminal law for children under 14	+	+	−	−	−	−	+	−	−	−	−	+	+	−	?	−	+	−	−	−
34 Abolition of tariffs	?	+	?	+	+	+	+	−	?	−	+	?	−	−	−	+	−	+	?	−
35 Second citizenship	−	−	+	+	+	+	−	+	+	+	?	−	+	+	+	+	+	+	−	?
36 Compulsory social year	+	?	−	−	−	−	+	−	−	−	+	?	−	−	+	−	+	−	−	+
37 Fossil fuels	+	+	−	−	−	−	+	+	+	−	−	−	+	+	−	−	+	−	−	+
38 Increase in the minimum wage	?	?	+	+	+	+	−	−	+	+	+	?	−	+	+	+	?	+	?	+

Source: [Bundeszentrale für politische Bildung 2025]

statistical variables whose proximity is characterized by their correlations. We replace missing answers by 0s to use correlation as a proximity measure, otherwise correlation fails to distinguishing between two different policy profiles when the parties equally answer to all questions except for those when one party abstains. In this case, the binary relation of pseudo identity $\text{corr}(x,y) = 1$ for policy profiles x,y turns out to be intransitive, creating certain inconsistencies. To illustrate this effect, let parties A,B,C have the following profiles:

A	B	C		A	B	C
+	+	+	$\longleftrightarrow$	1	1	1
-	-	-		-1	-1	-1
+	?	-		1	?	-1

If correlations between the columns are computed excluding the rows with missing values then

$$\begin{aligned}
\rho_{AB} &= \rho \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = 1 \\
\rho_{BC} &= \rho \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = 1 \\
\rho_{AC} &= \rho \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \end{pmatrix} = 0.5
\end{aligned}
\implies A \sim B \sim C \quad \text{but} \quad A \not\sim C.$$

If we replace the missing value ? by 0, the implications are more reasonable:

$$\begin{aligned}
\rho_{AB} &= \rho \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 1 & 0 \end{pmatrix} = 0.87 \\
\rho_{BC} &= \rho \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 0 & -1 \end{pmatrix} = 0.87 \\
\rho_{AC} &= \rho \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \end{pmatrix} = 0.5
\end{aligned}
\implies A \not\sim B \not\sim C \quad \text{and} \quad A \not\sim C.$$

Correlation ρ as a measure of ‘proximity’ (the higher the correlation, the higher the proximity), being inverted into $1 - \rho \geq 0$, is not a distance in the mathematical sense but only a vaguely understood pseudo distance. Of course, we could apply one of distances — Euclidean, Manhattan, Hamming, etc. However, we use correlation because it is standard for contiguously ordering statistical variables [Friendly 2002, Friendly and Kwan 2003].

2.2 Party ordering by votes received (V)

The correlation triangle in Figure 1 is the bottom-left half of the correlation matrix $\{\rho_{ij}\}$ for the parties’ policy profiles from Table 1. Here, the parties are ordered by the decreasing number of votes received in the 2025 election [Bundeswahlleiterin 2025]. This correlation triangle is a ‘relief table’ [Tangian 2011, p. 108], whose elements are colored like in geographical maps: high values are shown in brown as mountains, the moderately positive in green as plains, the moderately negative in pale blue as shallow waters, and strongly negative ones in dark blue — as deep ocean. For a contiguous party ordering, the following rule would hold: the closer to the triangle’s diagonal, the higher the correlation. Visually, if the parties were ordered contiguously then the profiles of neighboring parties would (highly) correlate and the brown mountains would build a ridge along the diagonal, having at their foot green plains, then pale blue shallow waters, and finally dark blue ocean depths in the bottom-left corner. Since the correlation

scope of this paper.

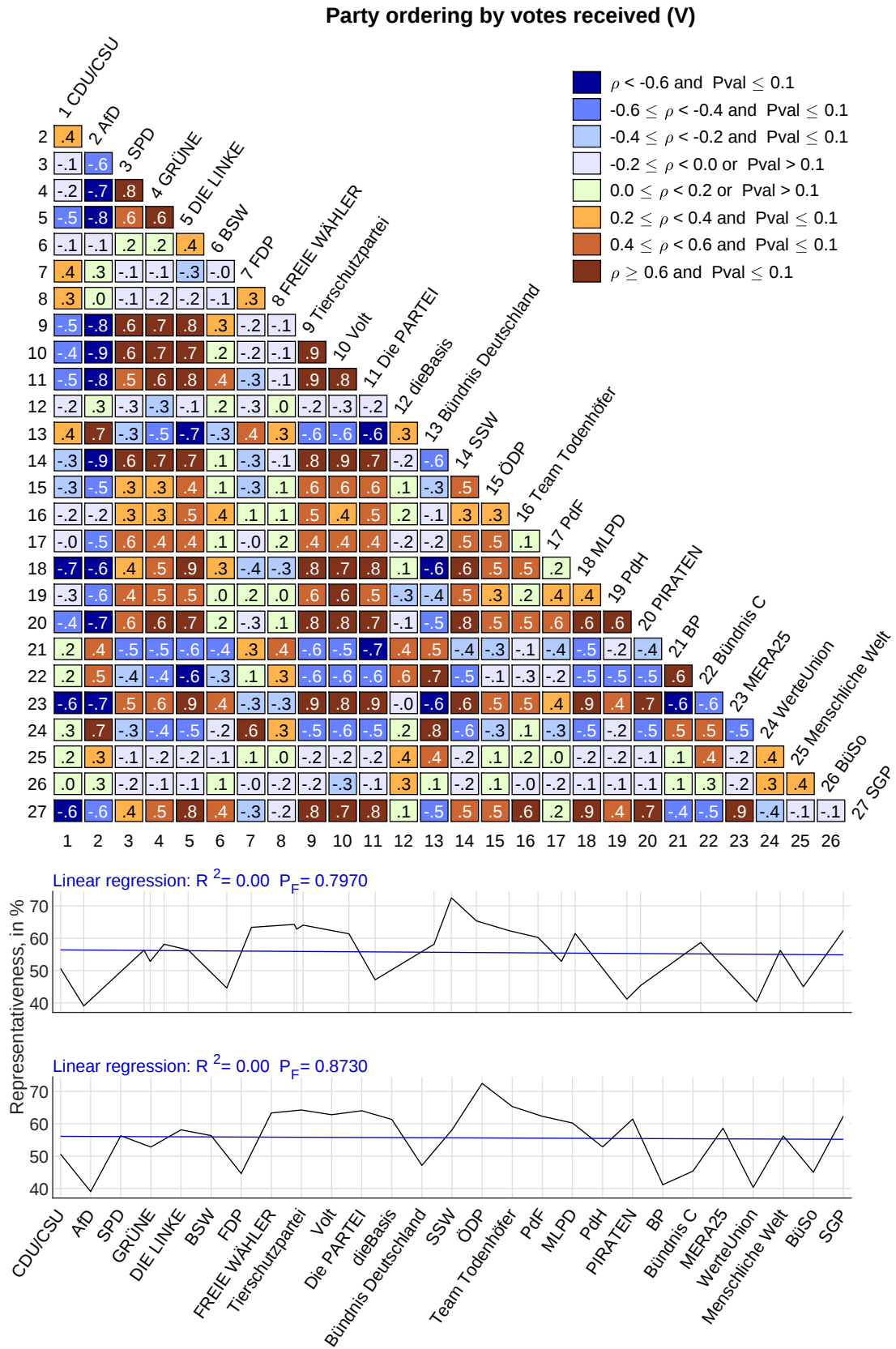


Figure 1: Correlation triangle for the 2025 policy profiles of German parties ordered by votes received (V). Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

triangle in Figure 1 lacks this structure completely, with colors scattered chaotically, the party ordering does not look contiguous.

The second and third plots of Figure 1 show the parties' representativeness (= mean of the parties' popularity and universality indices, both for unweighted and Google-weighted questions; see 'Mean index' in subscripts to blocks in [Tangian 2025b, Figure 2]). In the bottom plot, the distances between neighboring parties $i, i+1$ are assumed equal, which is reflected by the regular vertical grid. In the middle plot, the unequal distances between the grid lines are proportional to the 'distance' between the neighboring parties — inverted correlation $1 - \rho_{i, i+1}$, where $\rho_{i, i+1}$ are from the diagonal of the correlation triangle. As mentioned above, the correlation can be regarded as a proximity measure (the higher the correlation, the higher the proximity), and the inverted correlation $1 - \rho$ as a pseudo distance.

The horizontal blue regression lines in bottom plots of Figure 1 demonstrate the independence of the votes received by a party from its representativeness; see [Tangian 2025b, Table 4] and [Tangian 2025c, text description in Figure 1].

2.3 Party ordering by the representativeness index (R)

Figure 2 shows the correlations triangle for the parties ordered by decreasing representativeness (= mean of the parties' popularity and universality indices, both for unweighted and Google-weighted questions; see 'Mean index' in subscripts to blocks in [Tangian 2025b, Figure 2]). Naturally, the blue regression lines in two bottom plots of Figure 2 fit well to the curves of representativeness.

As one can see, the correlation triangles in Figures 1 and 2 lack the desired structure with brown 'mountains' along the triangle diagonal. There are neither parallel bands of green 'planes' nor blue 'waters' in the bottom-left corner. We conclude that neither votes received nor representative ability can be used to order parties in a contiguous way.

3 Political spectra obtained using dimensionality reduction

3.1 Principal component analysis (PCA)

In this section, the German political spectrum is constructed by means of Principal Component Analysis (PCA) invented by Karl Pearson [Pearson 1901]. PCA approximates a 'cloud of observations' — vectors in a multi-dimensional space — by an ellipsoid whose first diameter is directed along the observations' maximum variance, the second diameter is directed along the observations' second maximum variance, etc. These orthogonal diameters are new coordinate axes, and the first ones 'explain' most of the variance, so that other dimensions can be omitted without much loss of information. A principal component is the set of projections of the given vectors on the corresponding diameter. Since a principal component consists of observations' coordinates on the new axis, we, indeed, can speak of the observations' variance along each diameter. The new axes are linear combinations of the initial axes and are interpreted either as composite factors or just as a geometric characteristic of the set of observations. For introductions to PCA see [Husson et al. 2011, Jeong et al. 2009, Jackson 1988, Krzanowski 1988, Seber 1984].

First of all, we explain the idea of PCA by making a 2D map of a country, which, in actuality, is located on a 3D globe. Let n reference points, e.g. cities, be given as 3D coordinate vectors in the three-dimensional space. If the country is small, the least significant dimension associated with the earth's curvature is omitted, and only North-South and East-West dimensions are retained and reflected by the first and second principal components. However, the task is not that straightforward. For instance, in the case of Chile, which is a North-South strip 4250 km long and on average 180 km wide, the first component is associated with the North-South direction, the second with the earth's curvature, and the least significant third component with the East-West direction. Then the Chile map based on the first and second components would look like an arc — the side view of Chile on the globe — instead of the usual bird's-eye view. In fact, to make a map, we instead need to reflect the shortest air distances between the cities.

Party ordering by the mean representativeness (R)

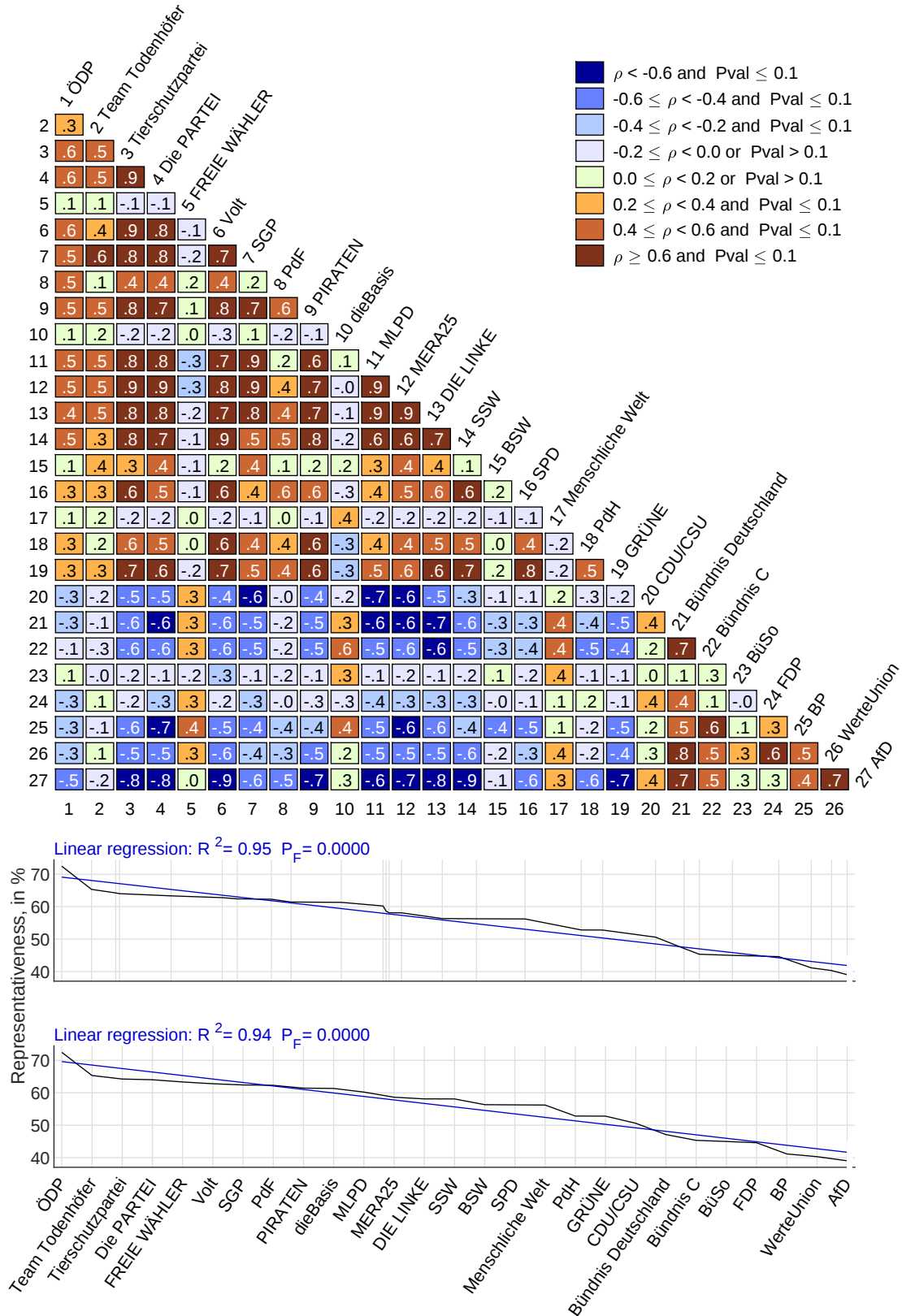


Figure 2: Correlation triangle for the 2025 policy profiles of German parties ordered by their mean representativeness index (R). Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

Therefore, we associate every city with a n -vector of its distances to other cities, including the 0-distance to itself, and apply the dimensionality reduction to the $(n \times n)$ -matrix of intercity distances rather than to the $(3 \times n)$ -matrix of 3D city spatial coordinates.

Since a political spectrum is a kind of map, its construction is very similar. The 27 German parties are analogous to cities, the (38×27) -matrix of the party profiles in Table 1 is analogous to the set of cities' spatial coordinates, and the (27×27) -matrix of inverted correlations $1 - \rho_{ij}$ is analogous to the distance matrix, to which PCA is applied. Since PCA is based on linear transformations, we apply it to the correlation (27×27) -matrix $\{\rho_{ij}\}$ with the same result as if it were applied to the matrix $\{1 - \rho_{ij}\}$.

Thus, the j -th party is identified with the so-called *party vector* with the j -th party's proximities to other parties, including the proximity to itself, that is, with the j -th column of the correlation (27×27) -matrix:

$$\vec{\rho}_j = \{\rho_{ij} : i = 1, \dots, 27\} \quad (\text{vector of the } j\text{th party}).$$

Thus, the party vectors are not the party profiles in Table 1 but the vectors of proximities to other party profiles. These 27-dimensional vectors, being considered as points in a 27-dimensional space, constitute at most a 26-dimensional configuration, and PCA finds its 26 orthogonal diameters — *eigenvectors* of the covariance matrix of the correlation matrix $\{\rho_{ij}\}$

$$\vec{e}_k, \quad k = 1, \dots, 26, \quad (\text{diameters of the 'cloud' of party vectors } \vec{\rho}_j)$$

and orders them by decreasing *eigenvalues*, which are the shares of the total variance. In this new orthogonal basis $\{\vec{e}_k\}$, each party vector $\vec{\rho}_j$ has its new coordinates $\{e_{kj}\}$:

$$\vec{\rho}_j \leftrightarrow \{e_{kj} : k = 1, \dots, 26\} \quad (\text{new coordinates of party vector } \vec{\rho}_j).$$

The first principal component is the set of the first coordinates of 27 vectors $\vec{\rho}_j, j = 1, \dots, 27$, in the new basis (projections of the 27 vectors $\vec{\rho}_j$ on $\vec{e}_1$):

$$\{e_{1j} : j = 1, \dots, 27\} \quad (\text{1st principal component with variance} = 88.76\%).$$

The second principal component is the set of the second coordinates of 27 vectors $\vec{\rho}_j, j = 1, \dots, 27$, in the new basis (projections of the 27 vectors $\vec{\rho}_j$ on $\vec{e}_2$):

$$\{e_{2j} : j = 1, \dots, 27\} \quad (\text{2nd principal component with variance} = 4.53\%),$$

and so forth. The projections of the parties' vectors $\vec{\rho}_j$ on the plane of the first two eigenvectors (largest diameters), covering 93.29% of the total variance, are shown in Figure 3.

3.2 Two-dimensional PCA solution (2D PCA)

Following [Friendly 2002, Friendly and Kwan 2003], we construct a contiguous party ordering using the first and the second principal components of the (27×27) -matrix of correlations $\{\rho_{ij}\}$ between the 38-dimensional policy profiles of 27 parties. The first and second principal components cover 93.29% of the total variance, providing a rather accurate representation of the space of party vectors (which elements are the inter-party proximities).

Figure 3 shows the projections of party vectors $\vec{\rho}_j$ on the plane of the first two principal components

$$\vec{\rho}_j = \{e_{1j}, \dots, e_{26j}\} \rightarrow \tilde{\vec{\rho}}_j = \{e_{1j}, e_{2j}\}, \quad j = 1, \dots, 27.$$

The angle between the j th party vector and the first eigenvector (X -axis) is equal to

$$\alpha_j = \begin{cases} \arctan\left(\frac{e_{2j}}{e_{1j}}\right) & \text{if } e_{1j} > 0 \\ \arctan\left(\frac{e_{2j}}{e_{1j}}\right) + \pi & \text{otherwise} \end{cases},$$

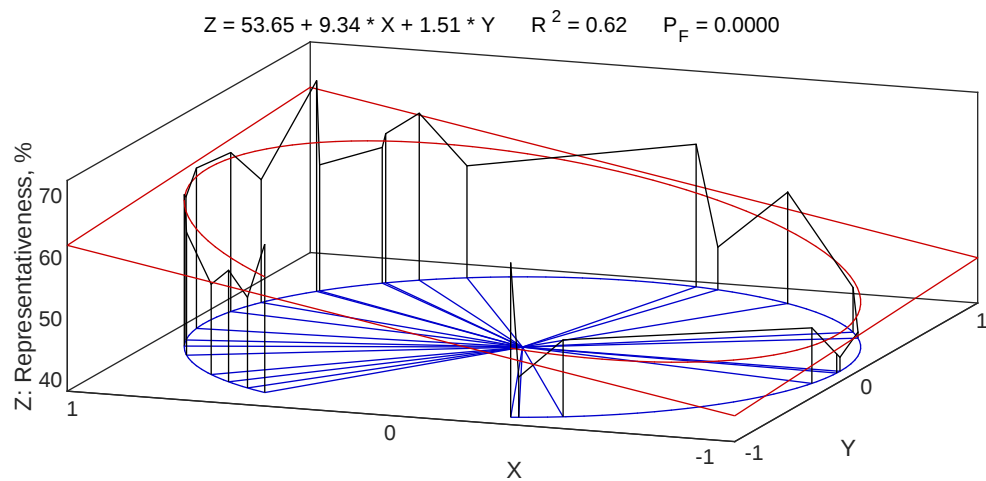
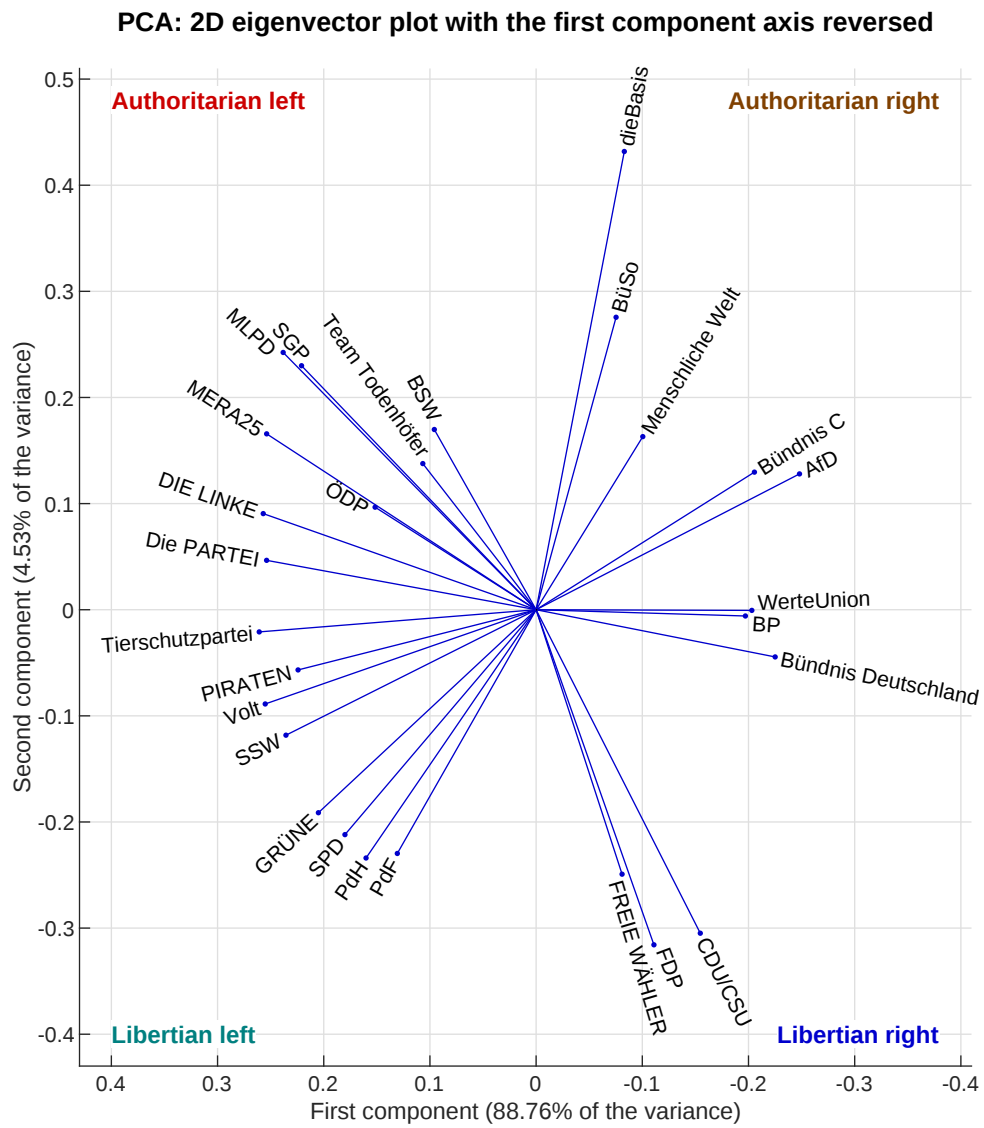


Figure 3: Principal component analysis of the correlation matrix for the 2025 policy profiles of German parties: the eigenvector and circular regression plots.

and the closeness of two parties' policy profiles is approximated by the angular closeness of the party vectors. To be precise, the correlation between profiles of two parties i, j is approximated by the cosine of the angle between their vectors in Figure 3:

$$\rho_{ij} \approx \cos |\alpha_i - \alpha_j|. \quad (1)$$

We obtain a circular ordering, where neighboring parties have close policy profiles. To reflect the parties' ideological orientation, the horizontal axis of the eigenvector plot is reversed, and its quadrants are correspondingly labeled. This circular ordering is unfolded to a linear one by cutting it at the largest gap — between the party vectors of CDU/CSU and Bündnis Deutschland. Figure 4 depicts the correlation triangle for the unfolded ordering with the desired structure — brown 'high mountains' along the diagonal, visualizing the ordering's contiguity. As seen in Figure 3, the party ordering is not rectilinear but horseshoe-like, which is also reflected in Figure 4: the correlation triangle's bottom-left elements are light brown, i.e., the ordering's left-hand and right-hand ends approach each other, remaining however somewhat distant.

As follows from (1), the angles between neighboring party vectors in Figure 3 represent their closeness only approximately. The 2D vectors in Figure 3 are projections of the original 27-dimensional proximity vectors. The length of the 2D projections indicates how close the 27-dimensional vectors are to the plane. If a 2D projection is long, then the party vector leans to the plane, meaning that it is well inscribed in the circular ordering. If a 2D projection is short, then the party vector sticks out prominently, meaning that its belonging to the circular ordering is more conditional. For example, MERA25 looks very close to ÖDP and not that close to DIE LINKE in Figure 3 but, in fact, MERA25 is much closer to DIE LINKE ($\rho = 0.9$) than to ÖDP ($\rho = 0.5$); see the correlation values in Figure 5. The most extreme deviation from the flat circular ordering is inherent in Team Todenhöfer whose vector has the shortest projection on the 2D plane. Therefore, it makes sense to cut the circular party ordering in Figure 3 not at the largest angular gap but at the point of the lowest correlation between the neighboring parties, that is, between dieBasis and BSW. We obtain the correlation triangle in Figure 5 with the party descriptions ordered respectively in Table 2.

The mathematical model reveals but the circular proximity of the parties, without identifying them as left-wing or right-wing. It is all the more surprising that the party ordering in Figure 5 begins with left-wing parties some of which, like SGP (Sozialistische Gleichheitspartei, Vierte Internationale = Socialist Equality Party, Fourth International) and MLPD (Marxistish-Leninistische Partei Deutschland = Marxist-Leninist Party of Germany), are classified as left-wing extremist by the German Office for the Protection of the Constitution; see Section 10. Next, there stand several socialist-oriented parties, then moderate and conservative parties ending with the AfD (Alternative für Deutschland = Alternative for Germany), whom the Federal Office for the Protection of the Constitution lists as a suspected case of right-wing extremist activities.

The circularity of the ordering means that there may be parties that fill in the gap between the right-wing and the left-wing. Indeed, after the left-right progression from Party 3, SGP, to Party 23, AfD, there are small little ideologized Parties 24–27, Bündnis C, Menschliche Welt, BüSo and dieBasis with populist slogans. The left Parties 1–2, BSW (Bündnis Sahra Wagenknecht — Vernunft und Gerechtigkeit = Sahra Wagenknecht Alliance — Reason and Justice) and Team Todenhöfer can also be regarded as transitional from the far-right wing to the far-left one, because of their eclectic mix of left political claims with conservative economic positions.

The two plots under the correlation triangle in Figures 4–5 are analogous to those in Figures 1–2. In the bottom plot, the distances between the parties are uniform, that is, the closeness of the party profiles is not taken into account, only the order. In the upper plot, the distances between the parties' ticks are made proportional to the angle between the party vectors in Figure 3, i.e., the closer the party profiles, the closer the ticks. The blue regression lines exhibit opposite decreasing/increasing trend in Figures 4–5 because of different unfolding of the circular party ordering.

The circularity of the ordering is considered using the circular regression model from [Tangian 2020,

2D PCA order cut at the largest angle between neighboring parties

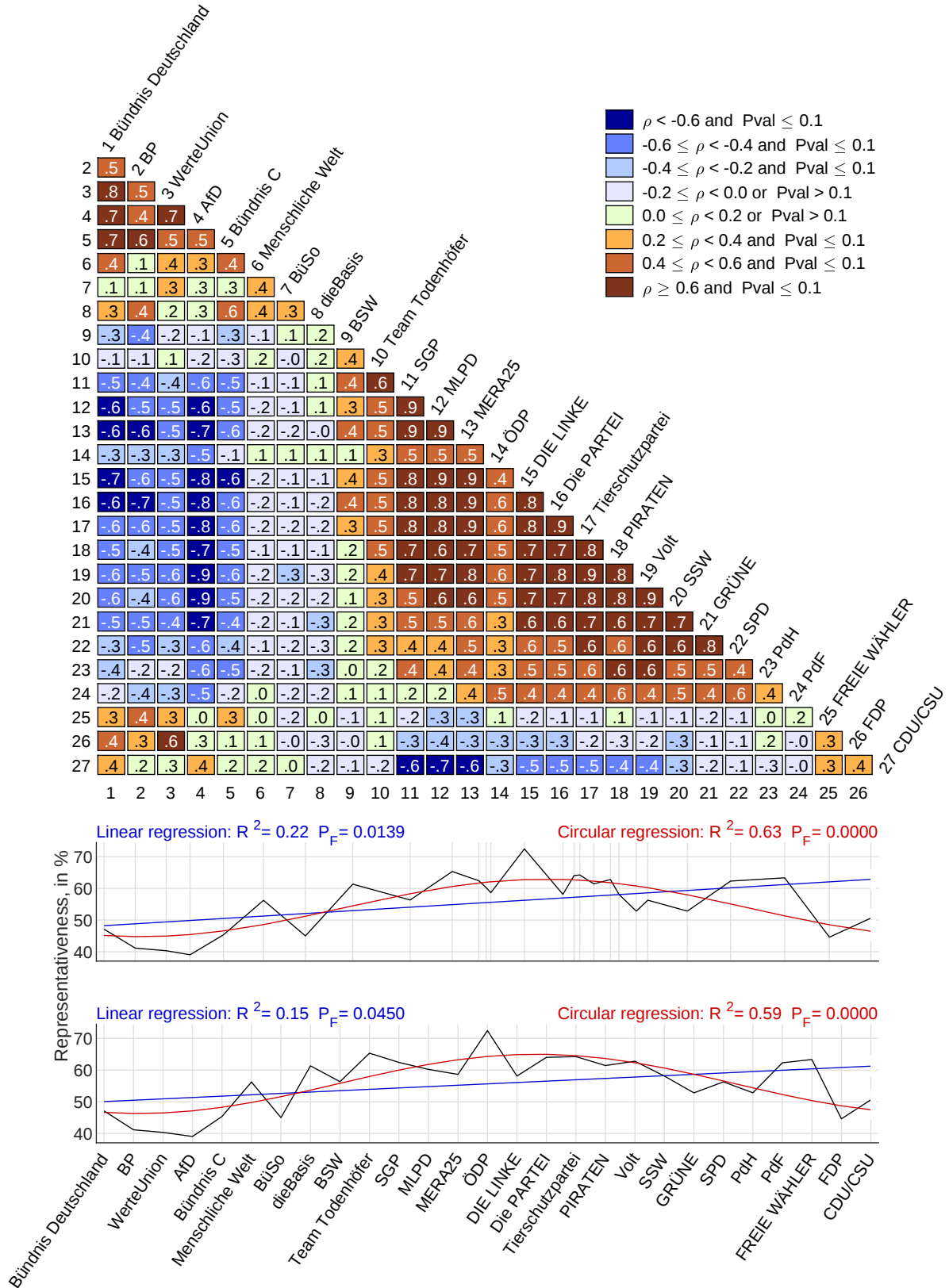


Figure 4: Correlation triangle for the 2025 policy profiles of German parties ordered by the 2D PCA model. The circular order is cut at the largest angle between the parties. Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

2D PCA order cut at the lowest correlation between neighboring parties

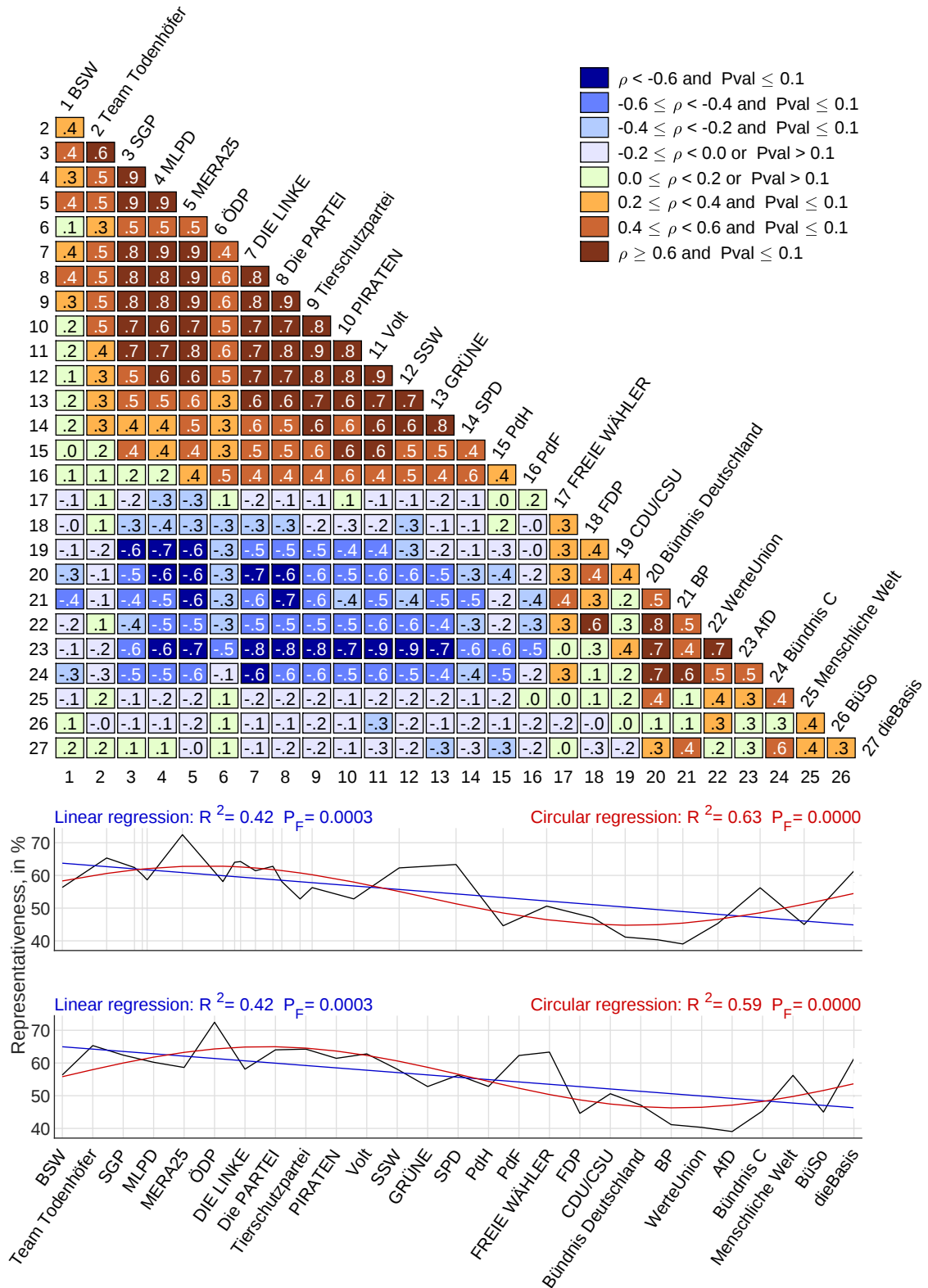


Figure 5: Correlation triangle for the 2025 policy profiles of German parties ordered by the 2D PCA model. The circular order is cut at the lowest correlation between neighboring parties. Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

Table 2: The ‘left–right’ ordering of German parties by the 2D PCA model

Party logo	Party description
1 	BSW, Bündnis Sahra Wagenknecht — Vernunft und Gerechtigkeit (Sahra Wagenknecht Alliance — Reason and Justice), was founded in 2024 by several members of the DIE LINKE party, among others, and is represented in the Bundestag through their transfer. Its namesake and co-founder is the Bundestag member and publicist Sahra Wagenknecht. The party combines economically left-wing content such as a wealth tax with socio-politically partly conservative positions, e.g. in migration policy.
2 	Team Todenhöfer, Die Gerechtigkeitspartei — Team Todenhöfer (The Justice Party — Todenhöfer’s Team) was founded in 2020 by former CDU MP Jürgen Todenhöfer. It campaigns for an end to the Bundeswehr’s foreign missions, is against national unilateral climate policy efforts and calls, among other things, for the construction of one million homes annually, a limitation of the term of office of MPs and a ban on party donations over 5,000 euros.
3 	SGP, Sozialistische Gleichheitspartei, Vierte Internationale (Socialist Equality Party, Fourth International), founded in 1971 as BSA, Bund Sozialistischer Arbeiter (Alliance of Socialist Workers) and called from 1997 to 2017 PSG, Partei für Soziale Gleichheit (Party of Social Equality), is a Trotskyist anticapitalist party. Its goals are the conquest of political power by the working class, the overthrow of capitalism and the “United Socialist States of Europe”. The Federal Office for the Protection of the Constitution classifies it as left-wing extremist.
4 	MLPD, Marxistisch-Leninistische Partei Deutschlands (Marxist-Leninist Party of Germany) is a communist party founded in 1982. It refers to Marx, Engels, Lenin, Stalin and Mao Zedong and sees itself as a radical left-wing alternative to other political forces. It is classified as left-wing extremist by the Federal Office for the Protection of the Constitution.
5 	MERA25 — Gemeinsam für Europäische Unabhängigkeit, (MERA25 — Together for European Independence) was founded first as Democracy in Europe (German: Demokratie in Europa) in 2020, and in 2021 was renamed to MERA25, in reference to the Greek party with the same name. The party is as part of the pan-European DiEM25. It calls for a radical change to a more directly democratic, more solidarity-based and more sustainable EU. Its goal is, among other things, the introduction of a “universal living income” and the socialization of key basic goods.
6 	ÖDP, Ökologisch-Demokratische Partei (Ecological Democratic Party) emerged from the ecology movement in 1981. Key issues include environmental and family policy, as well as democracy and the transparency of political processes. Since 2014, it has won one mandate in each of the European Parliament elections.
7 	DIE LINKE (The Left) was formed in 2007 through the merger of the PDS, Partei des Demokratischen Sozialismus (Party of Democratic Socialism), and the trade union-oriented WASG, Arbeit und soziale Gerechtigkeit — Die Wahlalternative (Labour and Social Justice — The Electoral Alternative). It is represented in the 2021 Bundestag. The Left advocates disarmament and the expansion of the welfare state, calls for a billionaire and wealth tax and wants to reduce the burden on small and medium incomes.
8 	Die PARTEI, Partei für Arbeit, Rechtsstaat, Tierschutz, Elitförderung und basisdemokratische Initiative (Party for Work, Rule-of-Law, Protection of Animals, Advancement of Elites and Grass-root Democratic Initiative) was founded in 2004. In its political work, it uses satirical means that it combines with serious core concerns such as the reduction of social inequality. Since 2014, it has won one and two mandates in the European Parliament elections.

Continued next page...

Table 2: (continued) The ‘left–right’ ordering of German parties by the 2D PCA model

Party logo	Party description
9  Tierschutzpartei	Tierschutzpartei: Mensch Umwelt Tierschutz (Animal Protection Party: People–Environment–Animal Protection) was founded in 1993. It is particularly committed to protecting the environment and animals. One of its goals is to include basic rights for animals in the Basic Law with a separate article. Since 2014, it has won one mandate in each of the European Parliament elections.
10 	PIRATEN, Piratenpartei Deutschland (Pirate Party of Germany) was founded in 2006 with a focus on internet policy issues such as data protection, digital copyright and net neutrality. They demand, among other things, a free and democratically controlled technical infrastructure and more powers for the federal government in education policy. From 2011 to 2017 they were represented in up to four state parliaments.
11 	Volt, Volt Deutschland (Volt Germany) was founded in 2018, is part of a pan-European movement. Important goals include a fundamental reform of the EU, extensive digitalization of administration and the switch to renewable energies. It has been a member of the European Parliament since 2019 — currently with three mandates.
12  Für uns im Norden.	SSW, Südschleswigscher Wählerverband (South Schleswig Association of Voters), was founded in 1948. It is the political lobby of the Danish minority and the Frisian ethnic group and is therefore exempt from the 5%-hurdle. Its focus is on northern Germany. Since 2021 the SSW is represented by one member of Bundestag.
13 	GRÜNE, BÜNDNIS 90/DIE GRÜNEN (Alliance 90/The Greens). The Greens were founded in 1980 and later joined forces with civil rights movements from the former GDR. They campaign for environmental protection, disarmament, renewable energies and gender equality, among other things. The Greens have been part of the federal government since 2021.
14 	SPD, Sozial-demokratische Partei Deutschlands (Social Democratic Party of Germany), founded in 1863. The SPD emerged from the workers’ movement in 1875. The slogan of social justice is the starting point for many of its positions, for example in the party’s labor, social and societal policies. Since the 2021 federal elections it the fourth time in its history that the party has a chancellor.
15 	PdH, Partei der Humanisten — Fakten, Freiheit, Fortschritt (Party of Humanists — Facts, Freedom, Progress). Founded in 2014, the PdH places people as self-determined, social and rational individuals at the centre of its politics. It opposes the connection between state and religion and wants to continuously adapt its positions to new scientific findings.
16 	PdF, Partei des Fortschritts (Party of Progress) was founded in 2020. It calls for more direct democratic elements. It finds its own positions in a grassroots democratic process. Among other things, it calls for a consistent energy transition and the expansion of public transport. In 2024, it won a mandate in the European Parliament.
17 	FREIE WÄHLER (Free Voters) was founded in 2009 as a nationwide association of local voter groups. They have been part of the Bavarian state government since 2018. The FREIE WÄHLER are calling for, among other things, a strengthening of local self-government, a simplification of tax law and a reduction in citizens’ allowances.
18 	FDP, Freie Demokratische Partei (Free Democratic Party), founded in 1948, emphasizes the freedom and self-determination of the individual. Among other things, it calls for a reduction in taxes and duties as well as the reduction of bureaucratic regulations and relies on innovations to tackle climate change. The FDP was part of the federal government until November 2024.

Continued next page...

Table 2: (continued) The ‘left–right’ ordering of German parties by the 2D PCA model

Party logo	Party description
19 	CDU/CSU, union of Germany’s main conservative parties, Christlich Demokratische Union Deutschlands (Christian Democratic Union of Germany) and Christlich-Soziale Union in Bayern (Christian Social Union of Bavaria). The CDU and CSU were founded in 1945–50 and 1945, respectively, as non-denominational parties and combine conservative, economically liberal and Christian-social positions. The CDU runs in elections in all federal states except for Bavaria, and the CSU runs in elections only there.
20 	BÜNDNIS DEUTSCHLAND (ALLIANCE GERMANY), also abbreviated to BD, founded in 2022, is considered right-wing conservative and economically liberal. The party focuses on individual achievement, personal responsibility and the strengthening of rural areas, among other things. Due to a party transfer, it is represented in the Bundestag by one member.
21 	BP, Bayernpartei (Bavaria Party) was founded in 1946. Its central goal are Bavarian statehood and the expansion of direct democracy. It also calls for the promotion of small and medium-sized businesses, the introduction of a childcare allowance and rejects dual citizenship.
22 	WU, WerteUnion (The Values Union) is a German party founded in 2024 by transforming a seven-year-old registered association with the same name. The party calls for, among other things, a restrictive migration policy and the abolition of public broadcasting in its current form. It wants to reduce the influence of the parties in favor of direct democratic elements. According to its own information, the Values Union had around 4,000 members in 2022; with about 3,000 also being the members of the CDU.
23 	AfD, Alternative für Deutschland (Alternative for Germany), was founded in 2013 and focuses, among other things, on restrictive positions in asylum and migration policy. It has been represented in the Bundestag since 2017. The Federal Office for the Protection of the Constitution lists it as a suspected case of right-wing extremist activities.
24 	Bündnis C, Bündnis C — Christen für Deutschland (Alliance C — Christians for Germany) was formed in 2015 from the merger of two Christian parties, PBC (Partei Bibeltreuer Christen = Party of Bible-abiding Christians) and AUF (Partei für Arbeit, Umwelt und Familie = Party of Labour, Environment and Family). The party derives positions such as the promotion of the traditional family, the rejection of abortion and the protection of the environment from Christian values and a biblical view of humanity.
25 	Menschliche Welt (The Human World) was founded in 2013. Its policies are based on a spiritual way of life and a decentralized common good economy. Issues such as peace policy, organic agriculture and economic justice play an important role in its program.
26 	BüSo, Bürgerrechtsbewegung Solidarität (Civil Rights Movement Solidarity), was founded in 1992 and sees itself as part of a movement that goes back to the late US political activist Lyndon LaRouche. It rejects man-made climate change as a hoax and warns of the collapse of the global financial system and Western civilization.
27 	dieBasis, Basisdemokratische Partei Deutschland (Basic Democratic Party of Germany) was founded in 2020 in the context of protests against the government measures to combat the Covid-19 pandemic. Among other things, it speaks out against the digitization of the health system. DieBasis also calls for the commissioning of ‘Nord Stream 2’ and referendums at all levels.

$$\beta_l = \frac{2\pi}{27}, \quad l = 1, \dots, 27. \quad (2)$$

As one can see, the circular regression provides much better quality of fit than the linear regression: it is characterized by much higher R^2 and negligible P_F . All of these mean a statistical dependence between the parties' left-right orientation and their representativeness, like for the years 2013 and 2017 [Tangian 2020, Chapters 9 and 14] but unlike to what was observed in 2021 [Tangian 2022b].

As one can see, the circular regression provides much better quality of fit than the linear regression: it is characterized by much higher R^2 and negligible P_F . All of these mean a statistical dependence between the parties' left-right orientation and their representativeness, like for the years 2013 and 2017 [Tangian 2020, Chapters 9 and 14] but unlike to what was observed in 2021 [Tangian 2022b].

In some cases, party vectors can be located along one predominant dimension. For instance, if all the correlations between party profiles are positive, or the party vectors in Figure 3 belong to a certain 90°-sector, then a contiguous party ordering can be obtained from projections of the party vectors on the first eigenvector:

Since our case is different, the party ordering by the first coordinates of the party vectors, as shown in Figure 6, is not contiguous. For example, the almost opposite 2D vectors of **Büso** and **FREIE WÄHLER** have close X -coordinates in Figure 3 and have therefore adjacent positions 17–18 in Figure 6; however, their correlation in Figure 6 is as low as -0.2 .

The two plots below the correlation triangle are analogous to the previous ones. The linear regression shows a certain dependence between the parties' left-right orientation and representativeness, and the more refined circular regression shows a certain preference for parties with a moderate socialist background. It should be however explained how the circular regression is adapted to the one-dimensional model. The linear ordering is rolled up, and the sinusoids are fit to the representativeness curves as described in the previous paragraph. To reflect the distance between the parties, the angles β_k between the adjacent party vectors and between the 27th and 1st party vectors in Figure 3 are made proportional to the inverted correlation coefficients $1 - \rho$, which are regarded as pseudo distances measures

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Party ordering by 1D PCA model

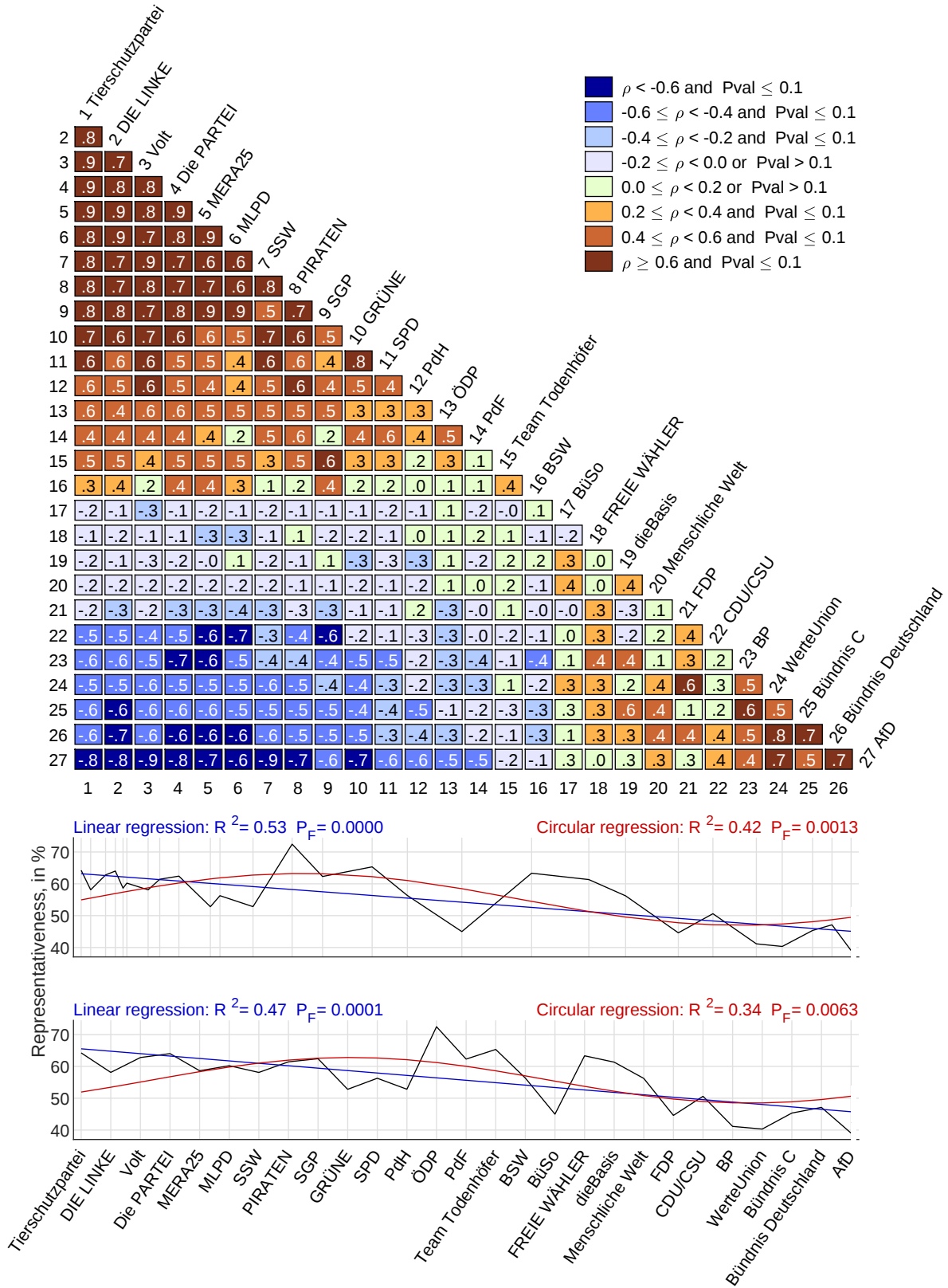


Figure 6: Correlation triangle for the 2025 policy profiles of German parties ordered by the 1D PCA model. Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

If the party proximity is not important, only the order, then the angles between the vectors of adjacent parties are assumed equal as in (2).

To conclude, the one-dimensional PCA model reflects the parties' left-right orientation in a very rough way. The more accurate model with two principal components reveals a left-right *progressive* party ordering and the *circularity* of the German political spectrum.

4 Left-right axis as a solution to the Traveling Salesman problem (TS)

Now we construct a circular party ordering by finding the shortest circular itinerary through the party profiles, making thereby the 'mountain ridge' along the correlation triangle's diagonal and enhancing its bottom left element. For this purpose, we reformulate the task in terms of the Traveling Salesman problem: given the intercity distance matrix for several cities, find the shortest cyclic itinerary through all of them, visiting each only once.

As before, we replace cities with party profiles and the distance matrix by the matrix of inverted correlations between them

$$\{d_{ij}\}, \quad \text{where} \quad d_{ij} = 1 - \rho_{ij}, \quad i, j = 1, \dots, 27.$$

Thus, we find the party ordering $\{i_k\}, k = 1, \dots, 27$, which minimizes the Traveling Salesman's (TS) total inverted correlation

$$\begin{aligned} TS &= d_{i_1 i_{27}} + \sum_{k=1}^{26} d_{i_k i_{k+1}} \\ &= 1 - \rho_{i_1 i_{27}} + \sum_{k=1}^{26} (1 - \rho_{i_k i_{k+1}}) \\ &= 27 - \rho_{i_1 i_{27}} - \sum_{k=1}^{26} \rho_{i_k i_{k+1}}. \end{aligned} \tag{4}$$

The upper plot of Figure 7 shows the shortest circular itinerary through the 27 parties, where the arcs are made proportional to $d_{i_k i_{k+1}}$ as in (3). The circular itinerary can be unfolded into a linear ordering by cutting one of its greatest arcs — between BŠo and ÖDP or between dieBasis and BSW, with the latter looking more adequate.

The correlation triangle for the unfolded counterclockwise party ordering is shown in Figure 8. With the only exception, the diagonal cells are brown, visualizing the high proximity of neighboring parties. The bottom-left edge is green, meaning that the ends of the party ordering approach each other. Nevertheless, the party ordering in Figure 8 looks rather inconsistent. In fact, the ordering falls into two segments: the first segment with the parties numbered from 1 to 16 can be characterized as a left–right progression, whereas the second segment with the parties numbers from 17 to 27 has, with reservations, an opposite 'ideological direction'. It appears that the Traveling Salesman model finds two short circular itineraries and joins them into one.

As in the previous figures, the two plots at the bottom show policy representation curves with and without taking into account the proximity of adjacent parties. The regression lines and regression sinusoids are fitted exactly in the same way as before; the plot at the bottom of Figure 7 illustrates the sinusoidal fit as in Figure 3. Both plots demonstrate a higher representativeness of the left-wing parties, with the regression lines and the regression sinusoids having greater values within the left segment of the party ordering.

Traveling salesman (TS): shortest circular itinerary with no longest segment

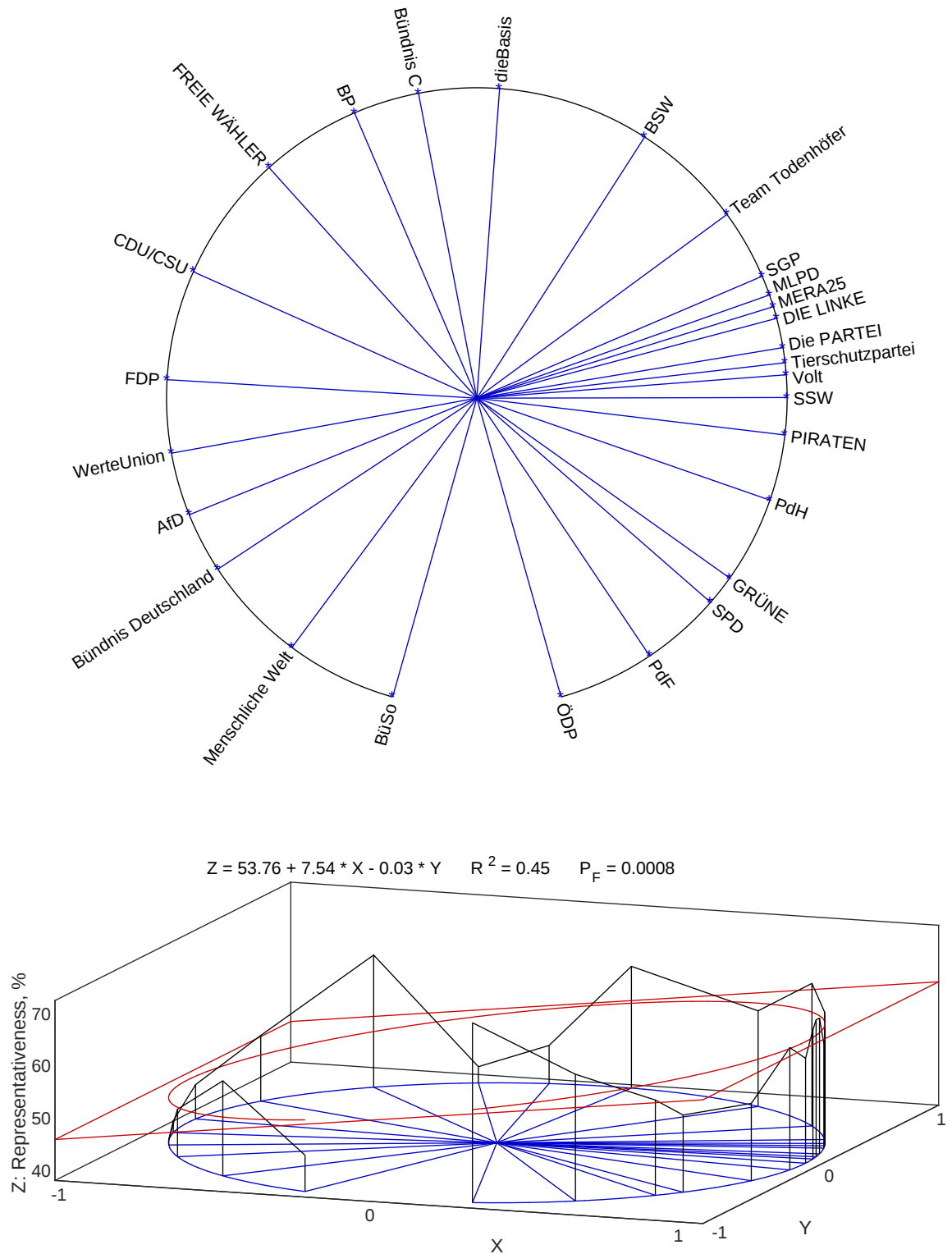


Figure 7: Traveling Salesman (TS) model application to the 2025 German party policy profiles: the shortest itinerary and the circular regression plot.

Party ordering as solution to Traveling salesman problem (TS)

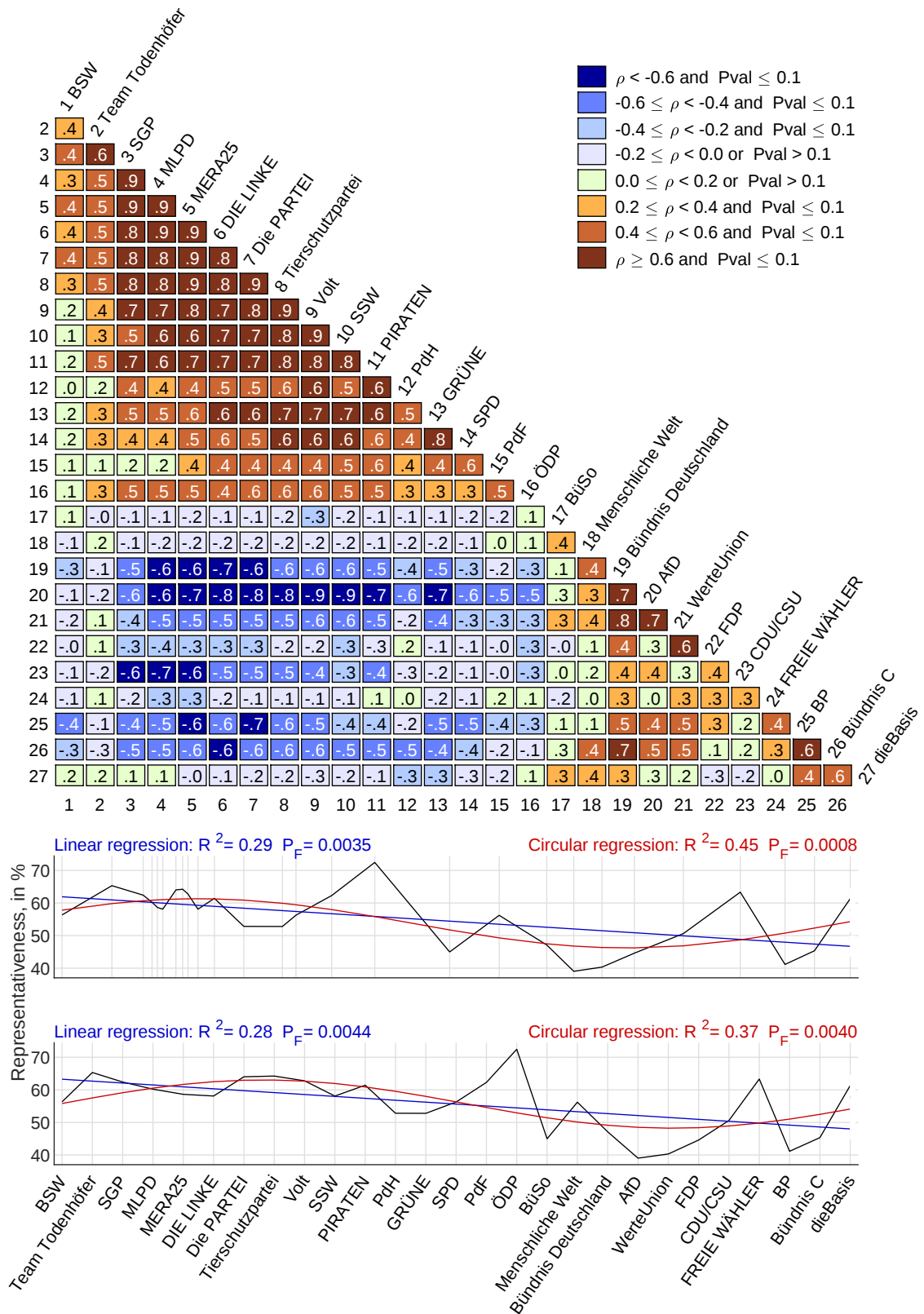


Figure 8: Correlation triangle for the 2025 policy profiles of German parties ordered by the Traveling Salesman model (*TS*). Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

5 Solutions using weighted squares criteria

In this section, we find contiguous party orderings using optimization criteria whose focus is larger than just the diagonal elements of the correlation matrix. These criteria take into account the overall dispersion of ‘heavy’ brown and ‘light’ blue elements.

5.1 Weighted least squares solution (ls)

First, we consider the weighted least squares criterion. We minimize ls — the weighted sum of squared Manhattan distances from the elements of the correlation triangle to its diagonal, with the weights being the correlation coefficients themselves. The Manhattan distance from an element to the diagonal is equal to the minimal number of steps to the diagonal:

$$\text{Manhattan distance of the } ij \text{ th cell to the diagonal} = |i - j| - 1 . \quad (5)$$

Hence, the optimization problem looks as follows:

$$ls = \sum_{i>j} (i - j - 1)^2 \rho_{ij} \rightarrow \min_{\text{Various party orderings}} . \quad (6)$$

If ‘heavy’ brown elements of the correlation triangle are located at the diagonal and ‘light’ blue elements are concentrated in the bottom-left corner then ls is small, and vice versa. Indeed, squared long distances multiplied by ‘heavier’ weights add too much to the ls value. Therefore, by minimizing ls , we move ‘heavy’ brown cells toward and ‘light’ blue cells away from the diagonal.

Figure 9 shows the resulting party ordering at the diagonal of the respective correlation triangle and two plots of policy representation analogous to those described earlier. We remind that the location of blue cells in the bottom-left corner of the correlation triangle means that the opposite parties are at the opposite ends of the party ordering.

To avoid exhaustive search when finding the party ordering that minimizes ls , we apply an iterative procedure, which we repeat until ls stops decreasing. In each iteration, we run a nested loop. In the main loop, we select parties one-by-one. In the inner loop, the party selected is relocated in the ordering to minimize the sum ls , testing all 27 alternative positions. The given application needed seven iterations.

5.2 Weighted largest squares solution (LS)

Now we apply the weighted largest squares criterion. We maximize LS — the weighted sum of squared Manhattan distance from the elements of the correlation triangle to its the bottom-left vertex, with the weights being the correlation coefficients themselves. By virtue of (5) and the observation that the Manhattan distance from the bottom-left cell to the diagonal is equal to 25, the optimization problem looks as follows:

$$\begin{aligned} LS &= \sum_{i>j} [25 - (i - j - 1)]^2 \rho_{ij} \\ &= \sum_{i>j} (26 - i + j)^2 \rho_{ij} \rightarrow \max_{\text{Various party orderings}} . \end{aligned} \quad (7)$$

The maximization algorithm is analogous to the one used to minimize the weighted squares in the previous subsection. Figure 10 shows the correlation triangle and two plots of policy representation for the party ordering found.

Party ordering as solution to weighted least squares problem (ls)

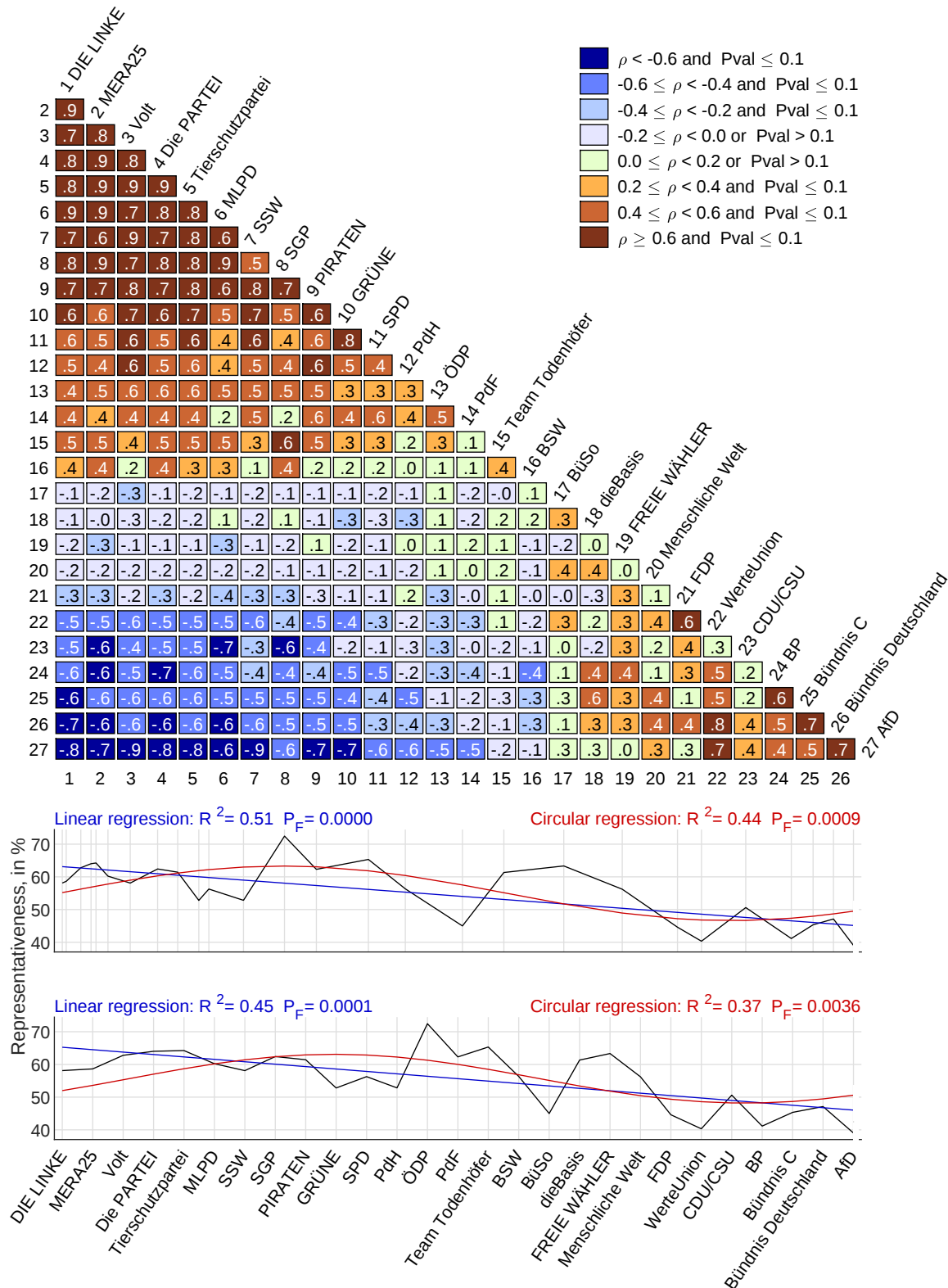


Figure 9: Correlation triangle for the 2025 policy profiles of German parties ordered by the weighted least squares (*ls*). Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

Party ordering as solution to weighted largest squares problem (LS)

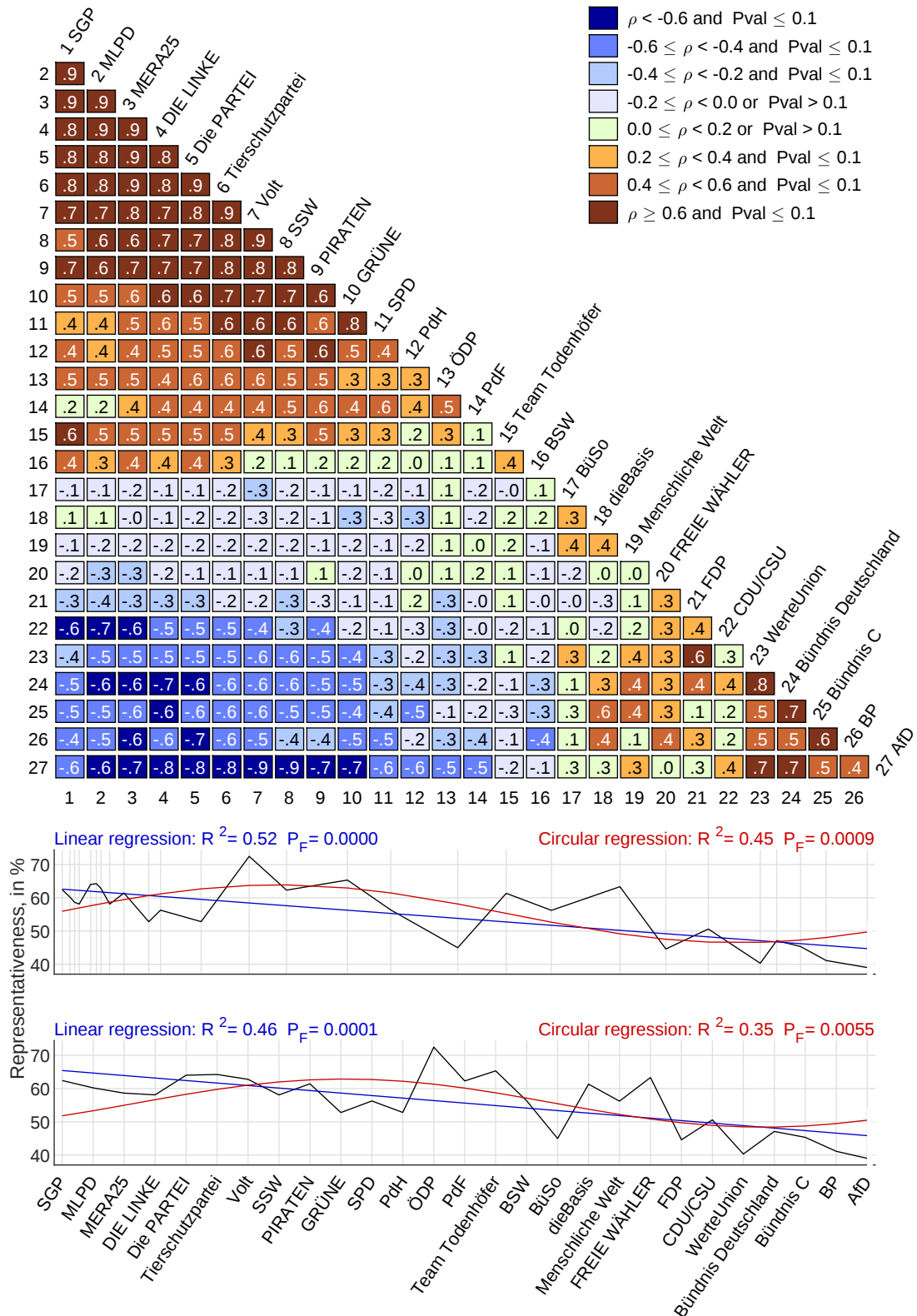


Figure 10: Correlation triangle for the 2025 policy profiles of German parties ordered by the weighted largest squares (LS). Two regression plots below are scaled with and without taking into account the correlation proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

Table 3: Spearman correlations of the 2025 German party ranks in 8 orderings and their evaluation by three scalar-valued criteria whose optima are framed

Ordering by	Votes	Representativeness	2D PCA angle cut	2D PCA corr cut	1D PCA	Traveling salesman	Least squares	Largest squares
Votes	1.00	0.01	−0.45*	0.09	0.05	0.16	0.00	−0.08
Representativeness	0.01	1.00	−0.33	0.65***	0.66***	0.57**	0.63***	0.64***
2D PCA angle cut	−0.45*	−0.33	1.00	−0.25	−0.43*	−0.54**	−0.39*	−0.36
2D PCA corr cut	0.09	0.65***	−0.25	1.00	0.71***	0.56**	0.71***	0.75***
1D PCA	0.05	0.66***	−0.43*	0.71***	1.00	0.70***	0.99***	0.95***
Traveling salesman	0.16	0.57**	−0.54**	0.56**	0.70***	1.00	0.68***	0.65***
Least squares	0.00	0.63***	−0.39*	0.71***	0.99***	0.68***	1.00	0.97***
Largest squares	−0.08	0.64***	−0.36	0.75***	0.95***	0.65***	0.97***	1.00
<i>Evaluation by criteria:</i>								
Traveling salesman (<i>TS</i>)	22.3	19.0	11.8	11.8	15.2	10.5	14.9	13.0
Least squares (<i>ls</i>)	1133	−5635	−4880	−6965	−12438	−7687	−12499	−12243
Largest squares (<i>LS</i>)	11370	22064	27378	29463	31551	29087	31722	32134
***	PVAL < 0.001							
**	0.001	<	PVAL	≤	0.01			
*	0.01	<	PVAL	≤	0.05			

6 Choosing the party ordering to be regarded as political spectrum

Table 3 shows the Spearman rank correlations between the eight party orderings considered so far: by votes received in the 2025 Bundestag elections, by the representativeness index — the mean of popularity and universality; see ‘Mean index’ under the blocks in [Tangian 2025b, Figure 2] — and the six orderings constructed in this paper. In the bottom section of the table, each party ordering is evaluated using scalar-valued criteria (4), (6) and (7), whose optima are framed.

As we have seen, the party orderings by votes received in elections or by representativeness are in no way contiguous and cannot be considered candidates for political spectrum. Other orderings are more or less contiguous, and we have to select the most appropriate one.

The ‘2D PCA corr cut’ ordering (the 2D PCA circular ordering cut at the lowest correlation) looks most credible because the 2D PCA model takes into account the joint spatial distribution of party vectors, the party order corresponds to established ideas, and because Row ‘2PCA corr cut’ in Table 3 consists of highest correlations on the average, which can be interpreted as consistency of several selection criteria. By obvious mathematical reasons, this ordering is more accurate than the 1D PCA ordering, which is restricted to one dimension instead of two.

The orderings obtained by the weighted least squares and weighted large squares models look quite reasonable and are similar, as follows from their very high correlation in Table 3. However, their high correlation with the 1D PCA ordering considered inferior to the 2D PCA corr cut ordering raises some concerns.

Finally, the ordering obtained using the Traveling Salesman model does not appears very consistent because the model focuses exclusively on adjacent party vectors neglecting surroundings, which is also confirmed by relatively low correlations of this ordering with others in Table 3.

Thus, the mathematical arguments and informal reasons speak in favor of the fact that the party ordering obtained using the 2D PCA corr cut model should be regarded as the German political spectrum.

7 Evolution of the German political spectrum

7.1 Uniting five political spectra in one

According to the conclusion of the previous section, the most appropriate model to construct the 2025 German political spectra — contiguous orderings of political parties — is the 2D PCA model. Applying it to the 2009, 2013, 2017 and 2021 *Wahl-O-Mat* data for the German federal elections, we obtain another four political spectra for the respective years. However, these spectra include different parties because only 13 parties out of totally 60 participated in all five elections and answered to the *Wahl-O-Mat* questions, which also differed from one election to another; for their description of 60 parties that at least once participated in the 2009–2025 elections see Section 10, ‘Appendix 2’.

Figure 11 displays the reshuffles in this ‘core’ of the German political spectrum constituted by the 13 parties that participated in all five elections considered. The red trajectories show the current Bundestag parties, and positions of other core parties are traced by green curves. As one can see, the left–right order of the Bundestag parties remains invariable except for the liberal FDP which, having failed in the 2013 elections, positioned itself in the 2017 elections even more conservative than the CDU/CSU but with no success. In the 2021 elections, it restored its liberal image and reentered the Bundestag.

The non-Bundestag parties, trying to win more votes, are less consistent and change their orientation more flexibly. For instance, the originally conservative Tierschutzpartei (Animal Protection Party) and FREIE WÄHLER (Free Voters) moved to the center. On the contrary, the rather left ÖDP (Ökologisch-Demokratische Partei = Ecological Democratic Party) and the rather right BüSo (Bürgerrechtsbewegung Solidarität = Civil Rights Movement Solidarity) have for a while adopted a more centrist air but then returned to their former positions.

In Figure 11, there are 13 positions for 13 parties and moving one position up or down implies permutations in the policy space. The situation is different in the case of varying spectra because all of them are of different size and moving to the left or to the right in the spectrum does not necessarily mean that the parties are reshuffled; see Figure 12. Some parties appear and disappear in the spectrum, shifting thereby the ranks of other parties in the ordering and complicating comparisons. The effect is analogous to what in social choice theory is known as ‘dependence on irrelevant alternatives’. For example, the SGP and MLDP had in 2021 the 2nd and 3rd leftist ranks respectively, which in 2025, without any reshuffles, became the 1st and 2nd just because the even more radical DKP did not participate in the 2025 elections.

To make comparisons, we need a common space to place there the five spectra, and a common (left-right) scale as its coordinate axis. To bring the five spectra to a ‘common denominator’, we order the 60 parties that appear in the 2009, 2013, 2017, 2021 and 2025 German federal elections basing on the five political spectra with 24, 29, 31, 37 and 27 parties contesting in these elections, respectively. In other words, we are going to construct a *united political spectrum* with 60 parties that is most close to the given five German political spectra. The task can be reformulated in terms of collective choice: given five individual preferences on five unequal but partially intersecting sets of alternatives (alternatives are political parties, and five political spectra are five individual orderings of the alternatives), find the collective preference on the united set of all alternatives (order all parties). To solve this problem, we apply the Condorcet and Borda methods (assumed well-known) with minor modifications.

7.2 Condorcet model

Let us represent each spectrum — a strict party ordering — by a *spectrum matrix*. Table 4 shows five matrices for five spectra in Figure 12 restricted to the six parties traced in red. In the 2009 spectrum, the FDP is more right-wing than the GRÜNE (see Figure 12), which is reflected by +1 in the (FDP, GRÜNE)-element of the table Section ‘2009 spectrum matrix’. In 2009, the FDP is more left-wing than the CDU/CSU, and this is reflected by –1 in the (FDP, CDU/CSU)-element. Since the AfD was established in February 2013, it is missing in the 2009 Spectrum, making the 2009-ordering incomplete,

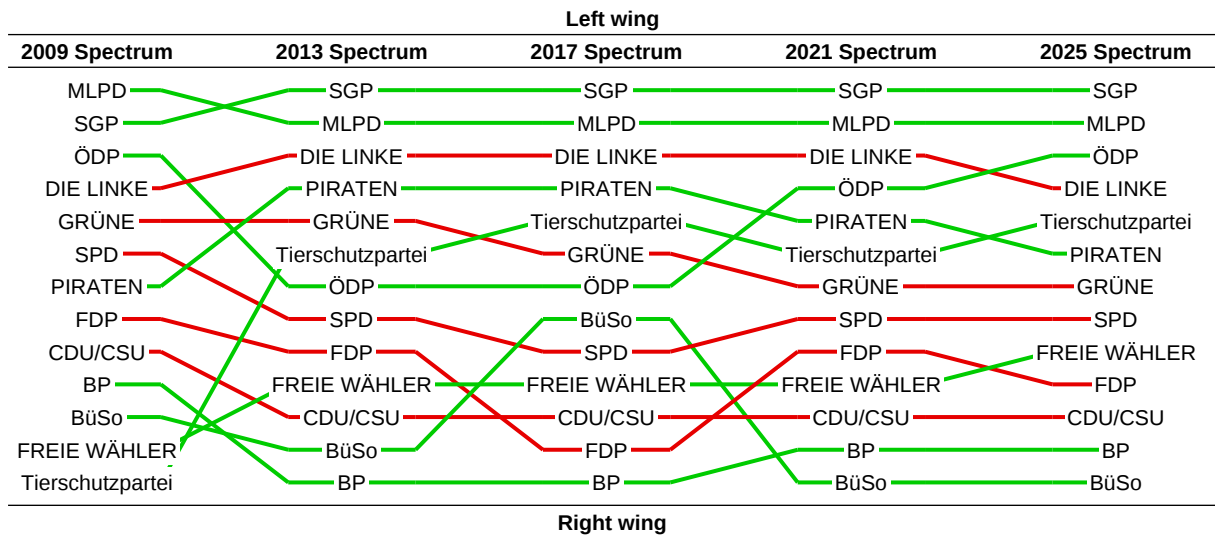


Figure 11: Reshuffles in the core of the German political spectrum. The parties represented in the Bundestag are traced in red. Other permanent participants in the German federal elections are traced in green.

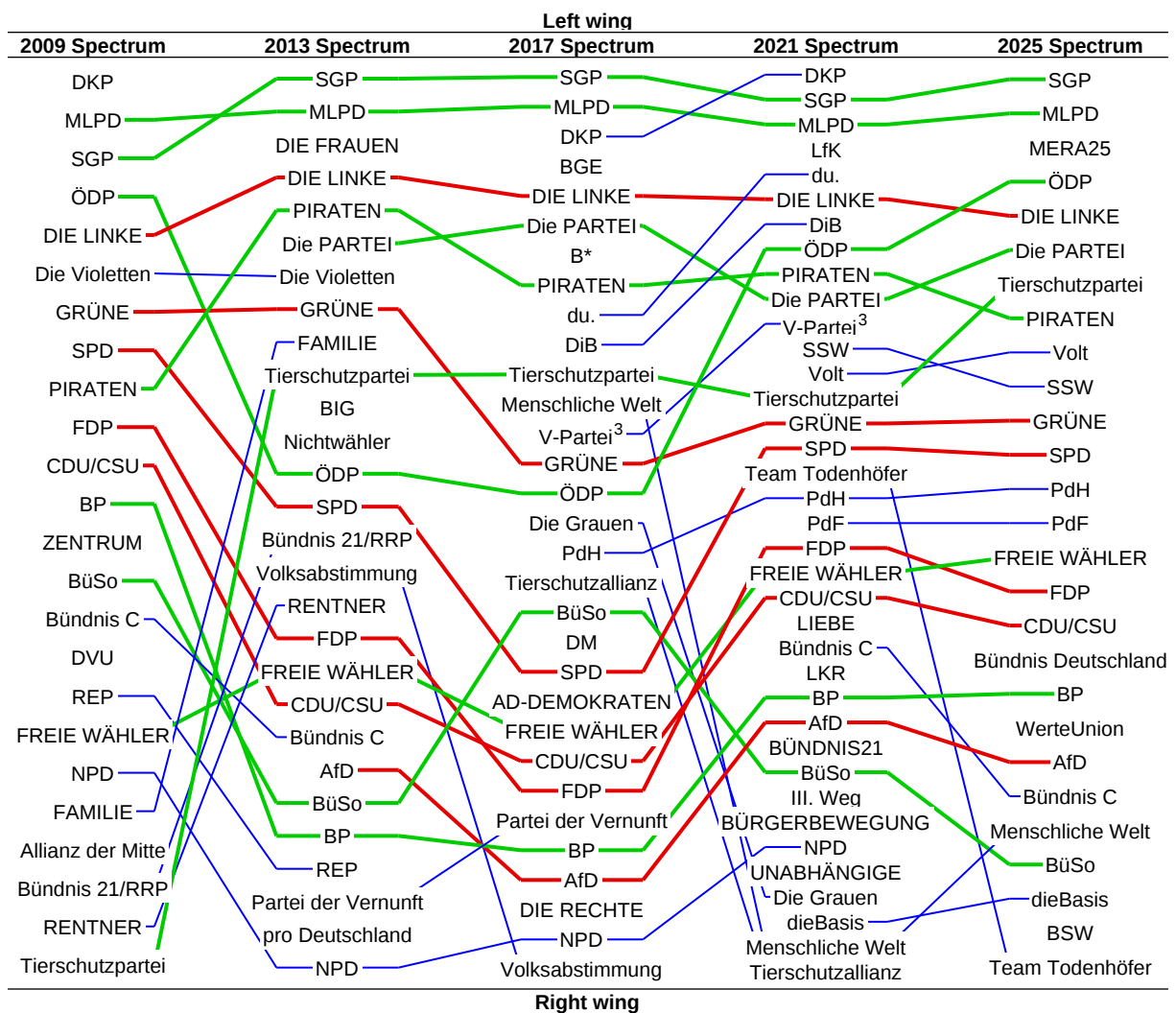


Figure 12: Reshuffles in the German political spectrum. The recent Bundestag parties are traced in red. Other permanent participants in the German federal elections are traced in green. Blue trajectories link the parties that took occasional participation in the Bundestag elections.

as reflected by missing elements in the AfD-column and the AfD-row.

Section ‘Sum of spectrum matrices’ of Table 4 shows the element-by-element sum of the five spectrum matrices above. Every its element is the score of the left-right relations between the respective parties over the period of 2009–2015 — the smaller (the ‘more negative’) the score, the more certain one party is more left than another.

For each party, the mean of its row in Section ‘Sum of spectrum matrices’ is shown in Column ‘Mean Left/Right-bias’. It specifies the relative left-right party orientation in terms of negative-positive values. These indices are ranked in the last column of Section ‘Sum of spectrum matrices’ with higher (= smaller) ranks corresponding to the left orientation. Here, DIE LINKE with the least mean L/R-bias gets the top Leftist-rank of 1, and AfD with its Mean L/R-bias of 3.33 gets the lowest Leftist-rank of 6 in the last column. According to this leftist-ranking, we obtain the following united spectrum (which includes AfD although it is missing in the 2009-spectrum):

$$\text{United political spectrum} = \text{DIE LINKE} \quad \text{GRÜNE} \quad \text{SPD} \quad \text{FDP} \quad \text{CDU/CSU} \quad \text{AfD} .$$

Sparse spectra with few parties can be given a higher weight assuming that the left-right distances between the parties are greater than in dense spectra with numerous parties. Therefore, we consider a weighted version of the model with spectrum weights that are inversely proportional to the number of parties in the spectrum. For the spectra with five or six parties, we define:

$$\text{Spectrum weights } w = 6/5 \quad 6/6 \quad 6/6 \quad 6/6 \quad 6/6 .$$

The sum of spectrum matrices with the above weights is shown in the bottom section of Table 4, ‘Weighted sum of spectrum matrices’. The united party ordering in the last column is the same as for the unweighted model, but, as we will see, this is not always the case.

For the full German spectra in Figure 12 with 24, 29, 31, 37 and 27 parties, the weights are as follows:

$$\text{Spectrum weights } w = 37/24 \quad 37/29 \quad 37/31 \quad 37/37 \quad 37/27 .$$

The model with weights will be called *Condorcet weighted*, as opposed to the unweighted version simply called *Condorcet*.

7.3 Borda model

The application of the Borda approach is very similar to the application of the Condorcet approach, differing in one single point. In Table 4 that implements the Condorcet approach, the elements of spectrum matrices are quasi Yes/No ($1/-1$) answers to dichotomous questions ‘Is Party i more right-wing than Party j ?’. In the very similar Table 5 for the Borda method (and the same reduced spectra with six parties), the elements of spectrum matrices are answers to questions ‘By how many positions in the spectrum is Party i more right-wing than Party j ?’; see relations between the red trajectories in Figure 12. For example, the FDP in the 2009 spectrum is more right-wing than the GRÜNE by two positions (focus on the red trajectories only!). Therefore, instead of $+1$ in Table 4, the corresponding element in Table 5 is equal to 2.

All operations with the spectrum matrices and in the two bottom sections of Table 5 — summation, weighted summation, taking means of the rows and ranking — are analogous to the operations in Table 4.

The model with weights will be called *Borda weighted*, as opposed to the unweighted version simply called *Borda*.

7.4 Mean Ranks model

The third model generalizes the Borda method. In the original Borda method, all individuals consider and rank the same set of alternatives, and then the alternatives are ordered by the sum of individual

Table 4: United party order computed using the Condorcet model

<i>i</i>	Is Party <i>i</i> more right-wing than Party <i>j</i> ?						Mean L/R-bias	L-Rank
	<i>j</i> : AfD	CDU/CSU	DIE LINKE	FDP	GRÜNE	SPD		
2009	AfD							
spectrum	CDU/CSU	0	1	1	1	1		
matrix	DIE LINKE	-1	0	-1	-1	-1		
weight = 6/5	FDP	-1	1	0	1	1		
	GRÜNE	-1	1	-1	0	-1		
	SPD	-1	1	-1	1	0		
2013	AfD	0	1	1	1	1		
spectrum	CDU/CSU	-1	0	1	1	1		
matrix	DIE LINKE	-1	-1	0	-1	-1		
weight = 6/6	FDP	-1	-1	1	0	1		
	GRÜNE	-1	-1	1	-1	0		
	SPD	-1	-1	1	-1	1		
2017	AfD	0	1	1	1	1		
spectrum	CDU/CSU	-1	0	1	-1	1		
matrix	DIE LINKE	-1	-1	0	-1	-1		
weight = 6/6	FDP	-1	1	1	0	1		
	GRÜNE	-1	-1	1	-1	0		
	SPD	-1	-1	1	-1	1		
2021	AfD	0	1	1	1	1		
spectrum	CDU/CSU	-1	0	1	1	1		
matrix	DIE LINKE	-1	-1	0	-1	-1		
weight = 6/6	FDP	-1	-1	1	0	1		
	GRÜNE	-1	-1	1	-1	0		
	SPD	-1	-1	1	-1	1		
2025	AfD	0	1	1	1	1		
spectrum	CDU/CSU	-1	0	1	1	1		
matrix	DIE LINKE	-1	-1	0	-1	-1		
weight = 6/6	FDP	-1	-1	1	0	1		
	GRÜNE	-1	-1	1	-1	0		
	SPD	-1	-1	1	-1	1		
Sum of	AfD	0	4	4	4	4	3.33	6
spectrum	CDU/CSU	-4	0	5	3	5	2.33	5
matrices	DIE LINKE	-4	-5	0	-5	-5	-4	1
	FDP	-4	-3	5	0	5	1.33	4
	GRÜNE	-4	-5	5	-5	0	-2.33	2
	SPD	-4	-5	5	-5	5	-0.67	3
Weighted	AfD	0	4	4	4	4	3.33	6
sum of	CDU/CSU	-4	0	5.20	3.20	5.20	2.47	5
spectrum	DIE LINKE	-4	-5.20	0	-5.20	-5.20	-4.13	1
matrices	FDP	-4	-3.20	5.20	0	5.20	1.40	4
	GRÜNE	-4	-5.20	5.20	-5.20	0	-2.40	2
	SPD	-4	-5.20	5.20	-5.20	5.20	-0.67	3

Table 5: United party order computed using the Borda model

i		By how many positions is Party i to the right of Party j ?					Mean L/R-bias		L-Rank
		j : AfD	CDU/CSU	DIE LINKE	FDP	GRÜNE	SPD		
2009	AfD								
spectrum	CDU/CSU		0	4	1	3	2		
matrix	DIE LINKE		-4	0	-3	-1	-2		
weight = 6/5	FDP		-1	3	0	2	1		
	GRÜNE		-3	1	-2	0	-1		
	SPD		-2	2	-1	1	0		
2013	AfD	0	1	5	2	4	3		
spectrum	CDU/CSU	-1	0	4	1	3	2		
matrix	DIE LINKE	-5	-4	0	-3	-1	-2		
weight = 6/6	FDP	-2	-1	3	0	2	1		
	GRÜNE	-4	-3	1	-2	0	-1		
	SPD	-3	-2	2	-1	1	0		
2017	AfD	0	2	5	1	4	3		
spectrum	CDU/CSU	-2	0	3	-1	2	1		
matrix	DIE LINKE	-5	-3	0	-4	-1	-2		
weight = 6/6	FDP	-1	1	4	0	3	2		
	GRÜNE	-4	-2	1	-3	0	-1		
	SPD	-3	-1	2	-2	1	0		
2021	AfD	0	1	5	2	4	3		
spectrum	CDU/CSU	-1	0	4	1	3	2		
matrix	DIE LINKE	-5	-4	0	-3	-1	-2		
weight = 6/6	FDP	-2	-1	3	0	2	1		
	GRÜNE	-4	-3	1	-2	0	-1		
	SPD	-3	-2	2	-1	1	0		
2025	AfD	0	1	5	2	4	3		
spectrum	CDU/CSU	-1	0	4	1	3	2		
matrix	DIE LINKE	-5	-4	0	-3	-1	-2		
weight = 6/6	FDP	-2	-1	3	0	2	1		
	GRÜNE	-4	-3	1	-2	0	-1		
	SPD	-3	-2	2	-1	1	0		
Sum of	AfD	0	5	20	7	16	12	10	6
spectrum	CDU/CSU	-5	0	19	3	14	9	6.67	5
matrices	DIE LINKE	-20.00	-19.00	0	-16.00	-5	-10.00	-11.67	1
	FDP	-7	-3	16	0	11	6	3.83	4
	GRÜNE	-16.00	-14.00	5	-11.00	0	-5	-6.83	2
	SPD	-12.00	-9	10	-6	5	0	-2	3
Weighted	AfD	0	5	20	7	16	12	10	6
sum of	CDU/CSU	-5	0	19.80	3.20	14.60	9.40	7	5
spectrum	DIE LINKE	-20.00	-19.80	0	-16.60	-5.20	-10.40	-12.00	1
matrices	FDP	-7	-3.20	16.60	0	11.40	6.20	4.00	4
	GRÜNE	-16.00	-14.60	5.20	-11.40	0	-5.20	-7.00	2
	SPD	-12.00	-9.40	10.40	-6.20	5.20	0	-2	3

Table 6: United party orderings computed using the Borda and Mean Ranks models

		Parties' leftism ranks in political spectra					Borda		Mean Ranks	
		2009	2013	2017	2021	2025	Sum	L-Rank	Mean	L-Rank
		$w = 6/5$	$w = 6/6$	$w = 6/6$	$w = 6/6$	$w = 6/6$				
Leftism ranks	AfD		6	6	6	6	24	5.5	6	6
	CDU/CSU	5	5	4	5	5	24	5.5	4.8	5
	DIE LINKE	1	1	1	1	1	5	1	1	1
	FDP	4	4	5	4	4	21	4	4.2	4
	GRÜNE	2	2	2	2	2	10	2	2	2
	SPD	3	3	3	3	3	15	3	3	3
Spectrum-weighted leftism ranks	AfD		6	6	6	6	24	5	6	6
	CDU/CSU	6	5	4	5	5	25	6	5	5
	DIE LINKE	1.2	1	1	1	1	5.2	1	1.04	1
	FDP	4.8	4	5	4	4	21.8	4	4.36	4
	GRÜNE	2.4	2	2	2	2	10.4	2	2.08	2
	SPD	3.6	3	3	3	3	15.6	3	3.12	3

ranks. In the *Mean Ranks* method, each individual considers and ranks only a subset of all alternatives (it is assumed that these subsets cover the set of all alternatives), and then the alternatives are ordered not by the sum but the *mean* of individual ranks. If all individuals deal with the same set of alternatives then the Mean Ranks method is equivalent to the Borda count (then the sums of individual ranks of alternatives are proportional to their means). The both methods are illustrated in Table 6.

The idea of referring to means instead of sums is to make the result (the mean) independent of the number of alternatives considered by each individual. The effect is seen in the upper section of Table 6. Due to missing AfD in the first row (Spectrum 2009 does not include AfD), the sum of four ranks of the AfD is equal to 24, the same as the sum of higher but more numerous ranks of the CDU/CSU, so AfD receives the same Leftist-rank as the CDU/CSU, which is most counterintuitive. If, instead of the sum, the mean of the row is considered then the AfD gets a higher Mean than the CDU/CSU, and, correspondingly, a lower Leftist-Rank than the CDU/CSU.

Applying this method to the 2009–2025 German political spectra, we construct the united spectrum. As for the two previous methods, we also consider a weighted version of the Mean Ranks model; then taking the mean is replaced by taking the weighted mean.

The model with weights will be called *Mean Ranks weighted* as opposed to the unweighted version simply called *Mean Ranks*.

7.5 United spectrum as a reference scale

As follows from Columns ‘L-Rank’ of Tables 4–6, the six united spectra obtained using the weighted and unweighted Condorcet, Borda and Mean Ranks models are the same. It is not the case of large united spectra derived from the complete German political spectra. These united spectra are shown in Figure 13, where the differences are traced by party trajectories. Most differences are due to displacements of small parties (connected by blue segments), whereas the major parties (highlighted in red and green) hold their positions more firmly. This can be explained by the fact that small parties, irregularly participating in elections, provide less information about their location in the political space, while different aggregation methods, like construction of united spectrum, may respond differently to the information deficiencies.

As already mentioned, we wish to use one of the six united spectra constructed as a common scale for the 2009–2025 German political spectra. To choose the most appropriate one, we stretch the five German political spectra onto each united spectrum and look where the total fitting error is minimum.

Figure 14 illustrates the idea of stretching the 2009 and 2013 German spectra on the united spectrum in the middle of the plot. The parties are connected by line segments, and the task is to minimize the vertical

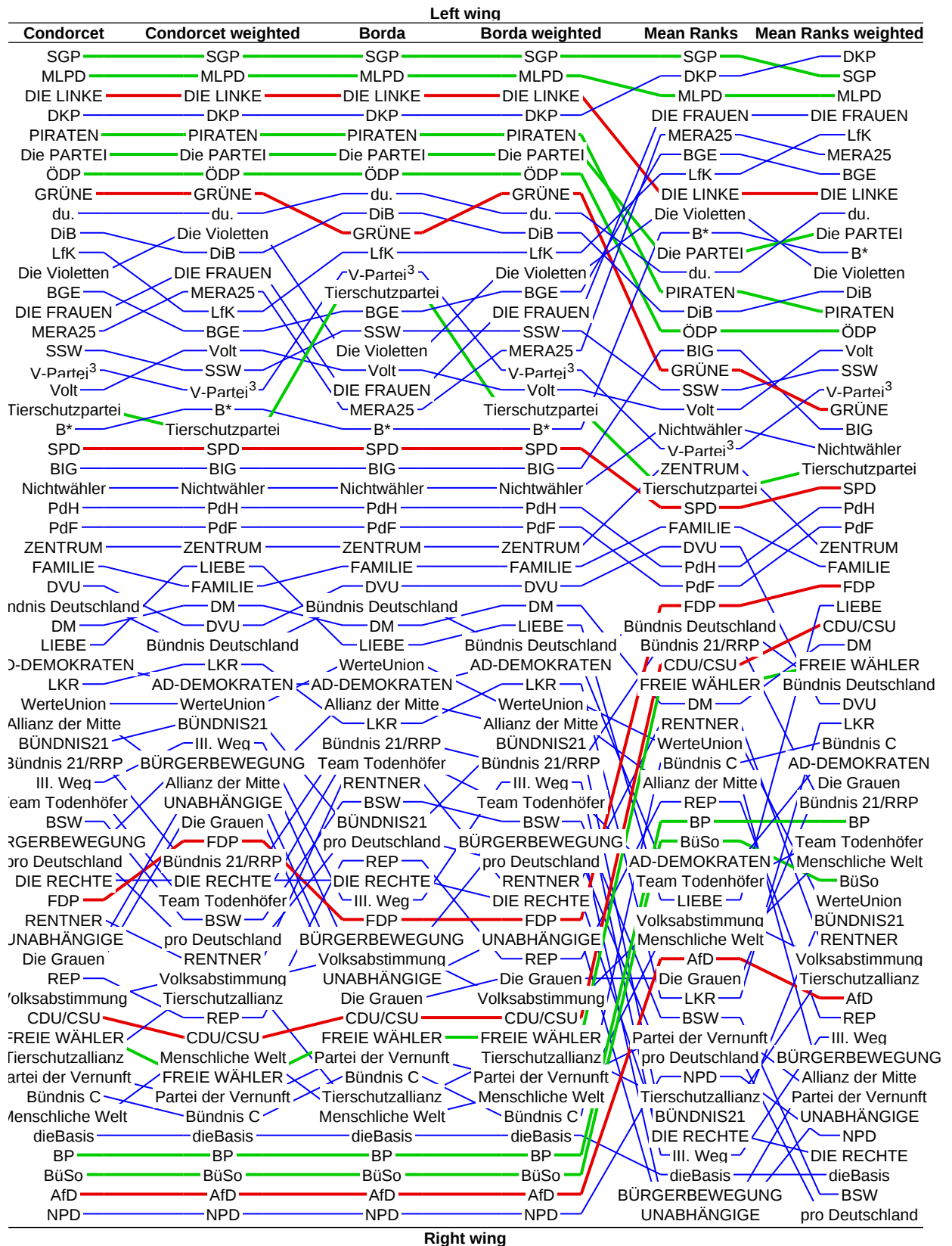


Figure 13: United political spectra obtained by six models

Table 7: Minimum total absolute and squared residuals for stretching five German political spectra on the united spectrum

Model	Total absolute residuals	Total squared residuals (in Ths)
Condorcet	973.3	11.52
Condorcet weighted	973.9	11.36
Borda	986.7	12.05
Borda weighted	978.2	11.54
Mean Ranks	586.6	5.03
Mean Ranks weighted	589.6	5.15

size of the connections, while preserving the party order in the 2009 and 2013 spectra. Considering the vertical size of connections as stretching (fitting) errors, we minimize the total of their squares. Stretching five German spectra on the united spectrum is done in the same way, and then the total stretching error embraces the stretching errors for all the five spectra. In other words, we reduce the problem to the standard least squares fitting, which is explained in mathematical detail in Subsection 7.6.

Table 7 shows the total stretching error for optimal fits of the 2009–2025 German spectra to the six universal spectra. Since the least stretching error is inherent in the united spectrum obtained using the unweighted Mean Ranks method, it is selected as the common scale to trace the changes in the 2009–2025 German political spectra.

The five German political spectra stretched on the united spectrum selected are shown in Figure 15. The united spectrum is not shown but it remains ‘behind the scene’, in particular determining the spatial arrangement of the 2009 and 2013 spectra — cf. Figures 14 and 15. Figure 15 shows the party dynamics, repositioning and grouping in the political space and time. The extreme left parties as well as moderate left ones constitute separate groups, the right-wing parties are also somewhat aside and the FDP runs always close to the CDU/CSU.

Among the major German parties highlighted in red, the SPD fluctuates by far the most between the left and the right. This political flexibility, or even inconsistency, can deter voters, especially floating voters without a firm self-identification with a particular party. All of these may explain the SPD’s failure in the 2025 elections, when the party received the historical minimum of 16.4% of the votes, having lost 9.3 percent points compared with the 2021 elections.

As for several small parties, they may sometimes look more extreme-right than AfD, III.Weg, REP and even NPD. In fact, they are rather intermediate between the extreme right and the extreme left, filling the populist left-right gap in the horse-shoe-shaped spectra we have constructed.

7.6 Mathematics of visualization

Now we stretch a German political spectrum S of size m on a United Spectrum U of size n , assuming that United Spectrum includes all parties from German spectrum. We enumerate the parties in United Spectrum U from the left-wing to the right-wing with integers from 1 to n and locate them along the vertical line axis, as shown in the middle of Figure 14, that is, these integers are vertical coordinates of the parties in United Spectrum (from top to bottom). Let the parties in German spectrum S be ordered from the left-wing to the right-wing and indexed with $i = 1, \dots, m$. By u_i we denote their positions in United Spectrum (their order does not correspond to the left-right orientation), defining thereby vector

$$\mathbf{u} = \{u_i, i = 1, \dots, m\}.$$

Stretching spectrum S on U means that the parties from S get new *real* coordinates x_i at the axis of United Spectrum while respecting the left-right order $x_i < x_{i+1}$ or, providing for line spaces between party names in the plots, $x_i + \varepsilon \leq x_{i+1}$. We collect these unknown coordinates in vector

$$\mathbf{x} = \{x_i, x_i + \varepsilon \leq x_{i+1}, i = 1, \dots, m\}$$

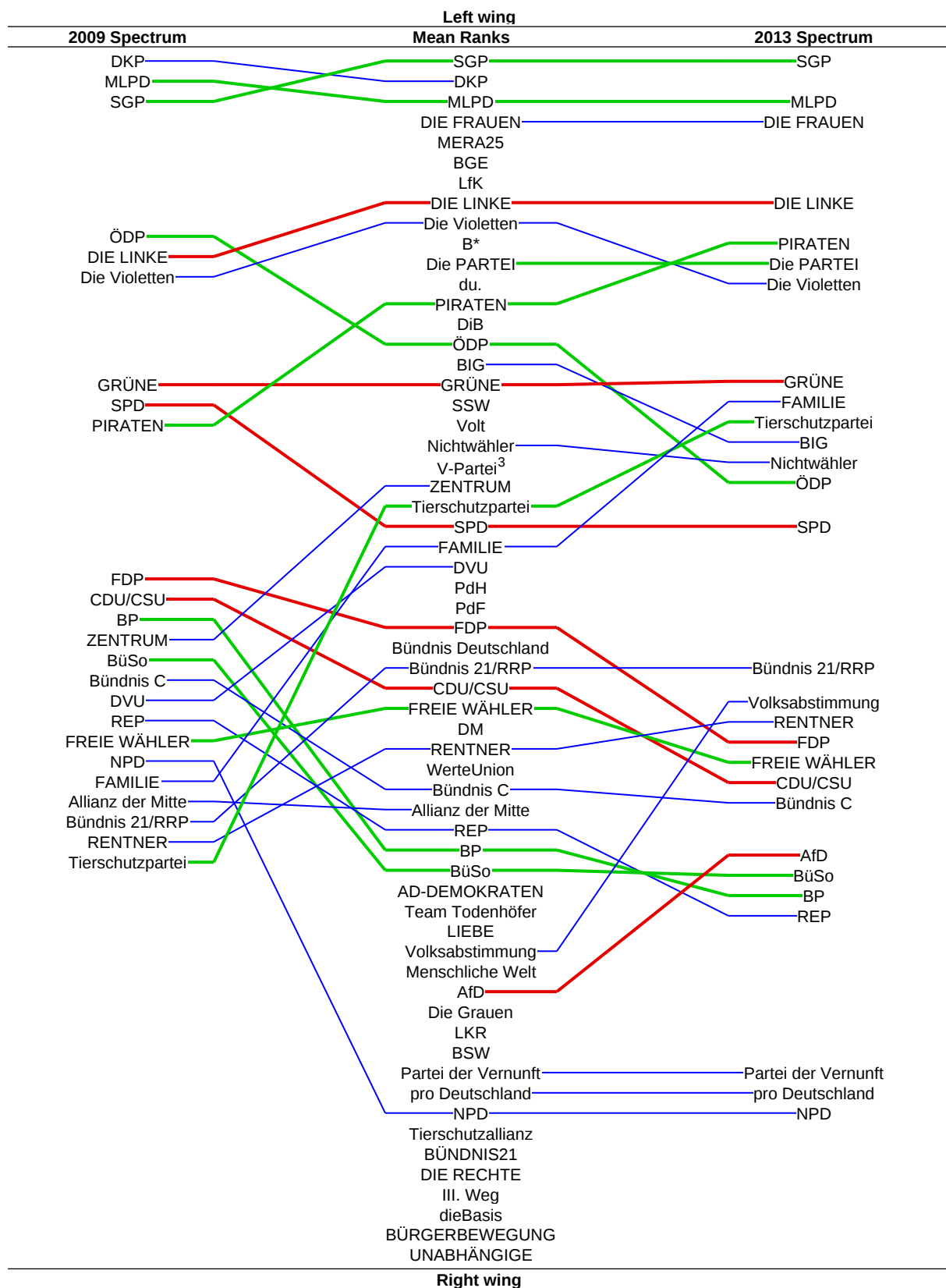


Figure 14: Stretching the 2009 and 2013 German spectra on the united spectrum obtained using the Mean Ranks method

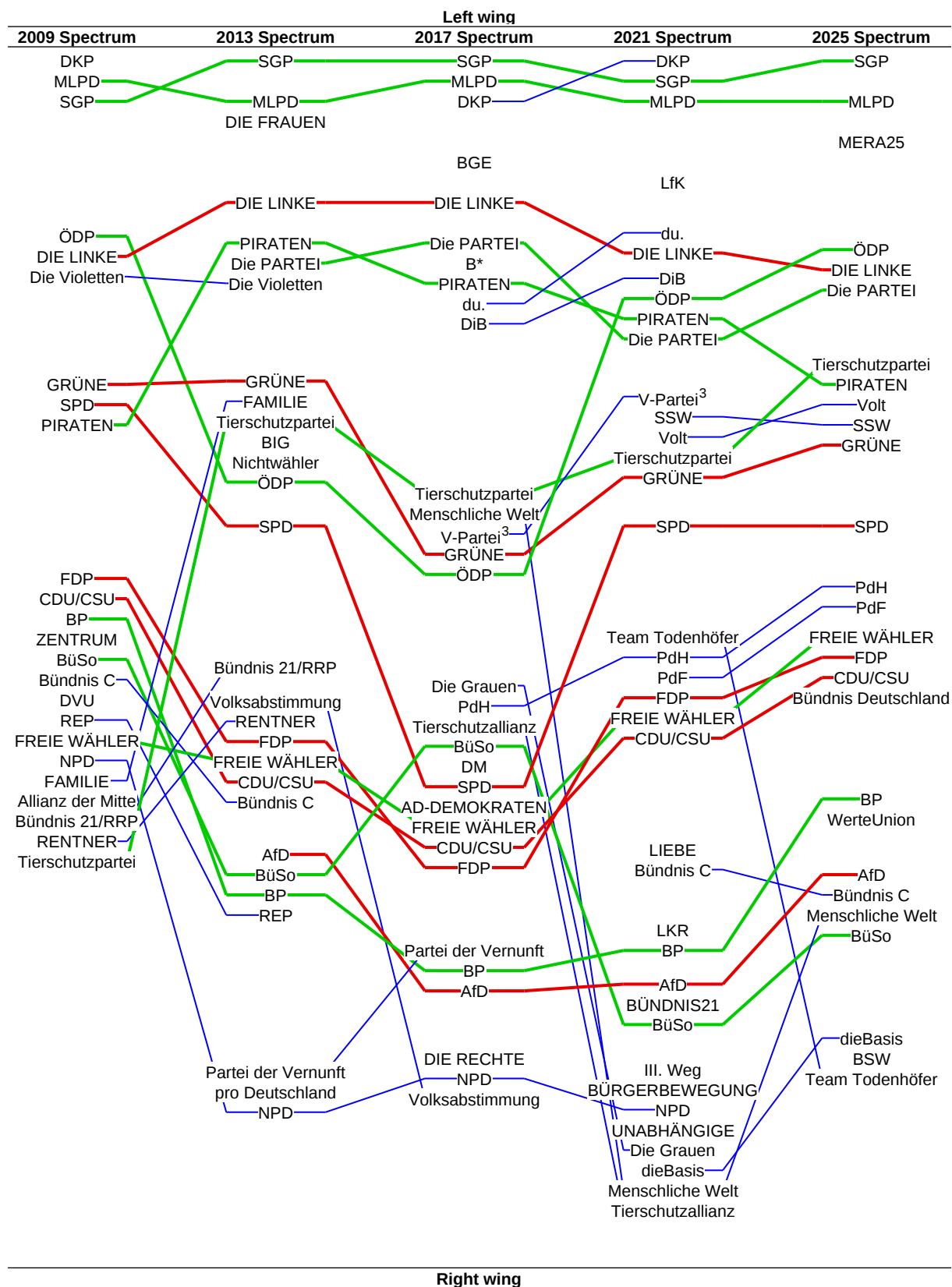


Figure 15: Reshuffles in the German political spectrum in 2009–2015 — visualization w.r.t. Mean Ranks method

and minimize the total squared fitting error $\|\mathbf{x} - \mathbf{d}\|^2$. In the standard notation, this linearly constrained least squares problem looks as follows:

$$\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{C} \cdot \mathbf{x} - \mathbf{u}\|_2^2 \quad \text{such that} \quad \mathbf{A} \cdot \mathbf{x} \leq \mathbf{b},$$

where

$$\mathbf{A} = \begin{pmatrix} -1 & 0 & \dots & 0 \\ 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & -1 \end{pmatrix} \quad \text{is } (m \times m)\text{-matrix}$$

$$\mathbf{b} = \varepsilon \cdot \begin{pmatrix} +\infty \\ -1 \\ \vdots \\ -1 \end{pmatrix} \quad \text{is } (m \times 1)\text{-vector with } \varepsilon > 0 \text{ being a small scalar}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix} \quad \text{is the identity } (m \times m)\text{-matrix}$$

The computations were performed using the MATLAB Optimization Toolbox function `lsqlin` for

$$\varepsilon = 1.$$

This way five German political spectra are stretched on the same United Spectrum. The quality of stretching of five German spectra is measured by the absolute or squared residual:

$$\text{Total stretching error} = \sum_{\text{Five spectra}} |\mathbf{C} \cdot \mathbf{x} - \mathbf{d}| \quad \text{or} \quad \sum_{\text{Five spectra}} \|\mathbf{C} \cdot \mathbf{x} - \mathbf{d}\|^2,$$

respectively; see Table 7 for the accuracy of optimal stretching of five spectra on six united spectra.

8 Conclusions

In this paper, we construct several versions of the 2025 German political spectrum, understood as a contiguous party ordering of the 27 German parties that participated in the 2025 federal elections. The contiguity is defined for the parties' policy profiles — 38-dimensional vectors of their Yes/No answers to 38 questions from the German voting advice application, the *Wahl-O-Mat*. The best result is obtained using the dimensionality reduction by the 2D Principal Component Analysis. In the final version, the German political spectrum looks as the left-right ideological axis rolled up in a horseshoe-shaped curve with the left-wing and the right-wing approaching each other, remaining however somewhat distant.

The one-dimensionality of the German political spectrum is an important prerequisite for consistent elections in Black's setting on single-peaked preferences along some common ordering of candidates. The existence of a common axis can explain, at least partially, why voting paradoxes are not observed in real-world elections as frequently as the theory predicts.

The left-right ideological alignments recognized by all the models considered call into question the assertion that the left-right characterization of parties is outdated. Through this paper, we — even if

indirectly — disagree with this viewpoint. The fact that parties find political niches close to the left-right axis means that the left-right orientation remains an important reference in political competition.

Having ensured the adequacy of the 2D PCA model, we apply it to construct the 2009, 2013, 2017 and 2021 German political spectra — for all years of the *Wahl-O-Mat* data availability. To make comparisons, we bring the five spectra to a ‘common denominator’ — construct a united spectrum for all 60 parties that at least once answered the *Wahl-O-Mat* questionnaire. For this purpose, we develop a model to order a set of alternatives basing on orderings of its subsets, and apply it to construct a united spectrum for 60 parties basing on the five political spectra found. Locating the five spectra in a space with the united spectrum as its reference axis, we trace the party dynamics, repositioning and grouping.

In particular, we discover that, among the major German parties, the SPD fluctuates by far the most between left and right. This political inconsistency can deter voters, especially floating voters without a firm self-identification with a particular party. All of these may explain the SPD’s failure in the 2025 elections, when the party received the historical minimum of 16.4% of the votes, having lost 9.3 percent points compared with the 2021 elections.

9 Appendix 1: The 2025 Wahl-O-Mat questions

1. **Wahl-O-Mat question:** *Unterstützung der Ukraine.* Deutschland soll die Ukraine weiterhin militärisch unterstützen.
English translation: *Support for Ukraine.* Germany should continue to support Ukraine militarily.
2. **Wahl-O-Mat question:** *Erneuerbare Energien.* Der Ausbau erneuerbarer Energien soll weiterhin vom Staat finanziell gefördert werden.
English translation: *Renewable energies.* The expansion of renewable energies should continue to be financially supported by the state.
3. **Wahl-O-Mat question:** *Streichung des Bürgergelds.* Das Bürgergeld soll denjenigen gestrichen werden, die wiederholt Stellenangebote ablehnen.
English translation: *Cancellation of the citizen's allowance.* The citizen's allowance should be cancelled for those who repeatedly turn down job offers.
4. **Wahl-O-Mat question:** *Tempolimit auf Autobahnen.* Auf allen Autobahnen soll ein generelles Tempolimit gelten.
English translation: *Speed limit on motorways.* A general speed limit should apply on all motorways.
5. **Wahl-O-Mat question:** *Abweisung Asylsuchender.* Asylsuchende, die über einen anderen EU-Staat eingereist sind, sollen an den deutschen Grenzen abgewiesen werden.
English translation: *Rejection of asylum seekers.* Asylum seekers who have entered the country via another EU country should be rejected at the German border.
6. **Wahl-O-Mat question:** *Begrenzung der Mietpreise.* Bei Neuvermietungen sollen die Mietpreise weiterhin gesetzlich begrenzt werden.
English translation: *Limitation of rental prices.* For new housing rentals, rental prices should continue to be limited by law.
7. **Wahl-O-Mat question:** *Automatisierte Gesichtserkennung.* An Bahnhöfen soll die Bundespolizei Software zur automatisierten Gesichtserkennung einsetzen dürfen.
English translation: *Automated facial recognition.* The Federal Police should be allowed to use software for automated facial recognition at train stations.
8. **Wahl-O-Mat question:** *Energieintensive Unternehmen.* Energieintensive Unternehmen sollen vom Staat einen finanziellen Ausgleich für ihre Stromkosten erhalten.
English translation: *Energy-intensive companies.* Energy-intensive companies should receive financial compensation from the state for their electricity costs.
9. **Wahl-O-Mat question:** *Rente nach 40 Beitragsjahren.* Alle Beschäftigten sollen bereits nach 40 Beitragsjahren ohne Abschläge in Rente gehen können.
English translation: *Pension after 40 years of contributions.* All employees should be able to retire without deductions after 40 years of contributions.
10. **Wahl-O-Mat question:** *Grundgesetz.* Im einleitenden Satz des Grundgesetzes soll weiterhin die Formulierung Verantwortung vor Gott stehen.
English translation: *Basic Law.* The introductory sentence of the Basic Law should continue to contain the phrase responsibility before God.
11. **Wahl-O-Mat question:** *Anwerbung von Fachkräften.* Deutschland soll weiterhin die Anwerbung von Fachkräften aus dem Ausland fördern.
English translation: *Recruitment of skilled workers.* Germany should continue to promote the recruitment of skilled workers from abroad.
12. **Wahl-O-Mat question:** *Nutzung der Kernenergie.* Für die Stromerzeugung soll Deutschland wieder Kernenergie nutzen.
English translation: *Use of nuclear energy.* Germany should use nuclear energy again to generate electricity.

13. **Wahl-O-Mat question:** *Anhebung des Spitzensteuersatzes.* Bei der Besteuerung von Einkommen soll der Spitzensteuersatz angehoben werden.
English translation: *Raising the top tax rate.* The top tax rate for income tax is to be raised.
14. **Wahl-O-Mat question:** *Kompetenzen in der Schulpolitik.* Der Bund soll mehr Kompetenzen in der Schulpolitik erhalten.
English translation: *Competencies in school policy.* The federal government should be given more competencies in school policy.
15. **Wahl-O-Mat question:** *Rüstungsexporte nach Israel.* Aus Deutschland sollen weiterhin Rüstungsgüter nach Israel exportiert werden dürfen.
English translation: *Arms exports to Israel.* Armaments should continue to be allowed to be exported from Germany to Israel.
16. **Wahl-O-Mat question:** *Krankenkassen.* Alle Bürgerinnen und Bürger sollen in gesetzlichen Krankenkassen versichert sein müssen.
English translation: *Health insurance companies.* All citizens should be insured in statutory health insurance companies.
17. **Wahl-O-Mat question:** *Abschaffung der Frauenquote.* Die gesetzliche Frauenquote in Vorständen und Aufsichtsräten börsennotierter Unternehmen soll abgeschafft werden..
English translation: *Abolition of the woman's quota.* The statutory quota for women on executive boards and supervisory boards of listed companies should be abolished.
18. **Wahl-O-Mat question:** *Ökologische Landwirtschaft.* Ökologische Landwirtschaft soll stärker gefördert werden als konventionelle Landwirtschaft.
English translation: *Organic farming.* Organic farming should be promoted more than conventional farming.
19. **Wahl-O-Mat question:** *Projekte gegen Rechtsextremismus.* Der Bund soll Projekte gegen Rechtsextremismus verstärkt fördern.
English translation: *Projects against right-wing extremism.* The federal government should increase its support for projects against right-wing extremism.
20. **Wahl-O-Mat question:** *Kontrolle von Zulieferern.* Unternehmen sollen weiterhin die Einhaltung der Menschenrechte und des Umweltschutzes bei allen Zulieferern kontrollieren müssen.
English translation: *Monitoring of suppliers.* Companies should continue to be required to monitor compliance with human rights and environmental protection by all suppliers.
21. **Wahl-O-Mat question:** *Elternabhängiges BAföG.* Die Ausbildungsförderung BAföG soll weiterhin abhängig vom Einkommen der Eltern gezahlt werden.
English translation: *Parent-dependent BAföG.* The BAföG (Bundes-Ausbildungs-Förderungs-Gesetz = Federal Training Support Act) training grant should continue to be paid depending on the parents income.
22. **Wahl-O-Mat question:** *Schuldenbremse.* Die Schuldenbremse im Grundgesetz soll beibehalten werden.
English translation: *Public debt brake.* The public debt brake in the Basic Law should be retained.
23. **Wahl-O-Mat question:** *Arbeitserlaubnis für Asylsuchende.* Asylsuchende sollen in Deutschland sofort nach ihrer Antragstellung eine Arbeitserlaubnis erhalten.
English translation: *Work permit for asylum seekers.* Asylum seekers in Germany should receive a work permit immediately after submitting their application.
24. **Wahl-O-Mat question:** *Verwerfen der Klimaziele.* Deutschland soll das Ziel verwerfen, klimaneutral zu werden.
English translation: *Abandoning climate targets.* Germany should abandon the goal of becoming climate neutral.
25. **Wahl-O-Mat question:** *35-Stunden-Woche.* In Deutschland soll die 35-Stunden-Woche als gesetzliche Regelarbeitszeit für alle Beschäftigten festgelegt werden.
English translation: *35-hour week.* In Germany, the 35-hour week is to be established as the legal standard working time for all employees.

26. **Wahl-O-Mat question:** *Schwangerschaftsabbruch nach Beratung.* Schwangerschaftsabbrüche sollen in den ersten drei Monaten weiterhin nur nach Beratung straffrei sein.
English translation: *Abortion after counseling.* Abortions in the first three months of pregnancy should continue to be legal only after counseling.
27. **Wahl-O-Mat question:** *Nationale Währung.* Der Euro soll in Deutschland durch eine nationale Währung ersetzt werden.
English translation: *National currency.* The euro is to be replaced by a national currency in Germany.
28. **Wahl-O-Mat question:** *Schiene vor Straße.* Beim Ausbau der Verkehrsinfrastruktur soll die Schiene Vorrang vor der Straße haben.
English translation: *Rail before road.* When expanding transport infrastructure, rail should have priority over road.
29. **Wahl-O-Mat question:** *Ehrenamt.* Ehrenamtliche Tätigkeiten sollen auf die zukünftige Rente angerechnet werden.
English translation: *Voluntary work.* Voluntary work should be credited towards future pensions.
30. **Wahl-O-Mat question:** *Umlegung der Grundsteuer.* Die Grundsteuer soll weiterhin auf Mieterinnen und Mieter umgelegt werden dürfen.
English translation: *Allocation of property tax.* Property tax should continue to be able to be allocated to tenants.
31. **Wahl-O-Mat question:** *Einschränkung des Streikrechts.* Das Streikrecht für Beschäftigte in Unternehmen der kritischen Infrastruktur soll gesetzlich eingeschränkt werden.
English translation: *Restriction of the right to strike.* The right to strike for employees in critical infrastructure companies should be restricted by law.
32. **Wahl-O-Mat question:** *Volksentscheide.* In Deutschland soll es auf Bundesebene Volksentscheide geben können.
English translation: *Referendums.* In Germany, referendums should be possible at the federal level.
33. **Wahl-O-Mat question:** *Strafrecht für unter 14-Jährige.* Unter 14-Jährige sollen strafrechtlich belangt werden können.
English translation: *Criminal law for children under 14.* Children under 14 should be able to be prosecuted.
34. **Wahl-O-Mat question:** *Abschaffung von Zöllen.* Deutschland soll sich für die Abschaffung der erhöhten EU-Zölle auf chinesische Elektroautos einsetzen.
English translation: *Abolition of tariffs.* Germany should campaign for the abolition of the increased EU tariffs on Chinese electric cars.
35. **Wahl-O-Mat question:** *Zweite Staatsbürgerschaft.* In Deutschland soll es weiterhin generell möglich sein, neben der deutschen eine zweite Staatsbürgerschaft zu haben.
English translation: *Second citizenship.* In Germany it should continue to be generally possible to have a second citizenship in addition to German citizenship.
36. **Wahl-O-Mat question:** *Soziales Pflichtjahr.* Für junge Erwachsene soll ein soziales Pflichtjahr eingeführt werden.
English translation: *Compulsory social year.* A compulsory social year should be introduced for young adults.
37. **Wahl-O-Mat question:** *Fossile Brennstoffe.* Neue Heizungen sollen auch zukünftig vollständig mit fossilen Brennstoffen (z. B. Gas oder Öl) betrieben werden dürfen.
English translation: *Fossil fuels.* New heating systems should continue to be allowed to run entirely on fossil fuels (e.g. gas or oil).
38. **Wahl-O-Mat question:** *Erhöhung des Mindestlohns.* Der gesetzliche Mindestlohn soll spätestens 2026 auf 15 Euro erhöht werden.
English translation: *Increase in the minimum wage.* The statutory minimum wage is to be increased to 15 euros by 2026 at the latest.

10 Appendix 2: The parties which participated at least in one of the 2009–2025 German federal elections

Table 8: The parties which participated in the German federal elections in 2009–2025

Party logo	Party description
1 	AD, Allianz Deutscher Demokraten (Alliance of German Democrats), founded in 2016, wants to enable people with immigrant background to live in Germany on the equal-rights basis. It is committed to dual citizenship and the rights of Muslim people, and is opposed to the EU in its present form.
2 	AfD, Alternative für Deutschland (Alternative for Germany), was founded in 2013 and focuses, among other things, on restrictive positions in asylum and migration policy. It has been represented in the Bundestag since 2017. The Federal Office for the Protection of the Constitution lists it as a suspected case of right-wing extremist activities.
3 	ADM, Alliance der Mitte (Alliance of the Centre), was a small German party that described itself as "bourgeois-conservative" and had regional associations in four federal states. In 2012, it merged with the German Conservative Party.
4 	B*, Bergpartei, die Überpartei (Mountain Party, Beyond-Party), founded in 2005. It is an alternative left-wing party with roots in the Berlin squatter scene. It calls for an unconditional basic income, pleads for restrictions on ownership, promotes exiting NATO and the direct exercise of political power by the people.
5 	BGE, Bündnis Grundeinkommen (Basic Income Alliance), founded in 2016. Its only political objective is implementing an unconditional basic income in Germany intended to enable everyone to participate in the community.
6 	BIG, Bündnis für Innovation und Gerechtigkeit (Alliance for Innovation and Justice), founded in 2010, a party of Muslims promoting their integration.
7 	BP, Bayernpartei (Bavaria Party) was founded in 1946. Its central goal are Bavarian statehood and the expansion of direct democracy. It also calls for the promotion of small and medium-sized businesses, the introduction of a childcare allowance and rejects dual citizenship.
8 	BSW, Bündnis Sahra Wagenknecht — Vernunft und Gerechtigkeit (Sahra Wagenknecht Alliance — Reason and Justice), was founded in 2024 by several members of the DIE LINKE party, among others, and is represented in the Bundestag through their transfer. Its namesake and co-founder is the Bundestag member and publicist Sahra Wagenknecht. The party combines economically left-wing content such as a wealth tax with socio-politically partly conservative positions, e.g. in migration policy.

Continued next page...

Table 8: (continued) The parties which participated in the German federal elections in 2009–2025

	Party logo	Party description
9		Bündnis 21/RRP, Bündnis 21 / Rentnerinnen- und Rentner-Partei (Alliance 21 / Party of Retirees), founded in 2007, promoting improving the pension, health and education systems.
10		Bündnis C, Bündnis C - Christen für Deutschland (Alliance C party) is founded in 2015 from the merger of two Christian fundamentalist parties. It advocates the promotion of traditional family models and wants to preserve creation in the sense of her Christian understanding of politics.
11		BÜNDNIS DEUTSCHLAND (ALLIANCE GERMANY), also abbreviated to BD, founded in 2022, is considered right-wing conservative and economically liberal. The party focuses on individual achievement, personal responsibility and the strengthening of rural areas, among other things. Due to a party transfer, it is represented in the Bundestag by one member.
12		BÜNDNIS21, diePinken/BÜNDNIS21 (the Rose/Alliance21) unites various small parties and political groups and is founded in early 2021. The party sees itself as the liberal-conservative political center and relies on a functioning constitutional state, the self-determination of the individual and the social market economy.
13		BÜRGERBEWEGUNG, Bürgerbewegung für Fortschritt und Wandel (Citizens' movement for progress and change), is founded in 2021. Above all, it advocates more direct citizen participation. It also wants to support small and medium-sized companies, among other things, and demands that employees benefit more from digitization.
14		BüSo, Bürgerrechtsbewegung Solidarität (Civil Rights Movement Solidarity) is founded in 1992 and sees itself as part of a movement that goes back to the US political activist Lyndon LaRouche, who died in 2019. It warns of the collapse of the global financial and economic system and advocates increased cooperation with China and Russia.
15		CDU/CSU, union of Germany's main conservative parties, Christlich Demokratische Union Deutschlands (Christian Democratic Union of Germany) and Christlich-Soziale Union in Bayern (Christian Social Union of Bavaria). The CDU and CSU were founded in 1945–50 and 1945, respectively, as non-denominational parties and combine conservative, economically liberal and Christian-social positions. The CDU runs in elections in all federal states except for Bavaria, and the CSU runs in elections only there.
16		DiB, Demokratie in Bewegung (Democracy in Motion), founded in 2017. DiB calls for greater co-determination, transparency in politics and the introduction of binding lobbyists. It supports the expansion of the European Union's competences and migration with human rights.
17		DIE FRAUEN (The Women), a feminist party founded in 1995, promoting the rights of women.
18		Die Grauen — Für alle Generationen (The Grays — For All Generations) is established in 2017. It is committed to social justice and calls, among other things, for more citizen participation, referendums at the federal level and a lowering of the voting age to 14 years. Despite its name, the party does not see itself as a pure advocacy group for older people. It deals also with strengthening of direct democracy and reduction of the five per cent hurdle for parties in representative bodies.
19		DIE LINKE (The Left) was formed in 2007 through the merger of the PDS, Partei des Demokratischen Sozialismus (Party of Democratic Socialism), and the trade union-oriented WASG, Arbeit und soziale Gerechtigkeit — Die Wahlalternative (Labour and Social Justice — The Electoral Alternative). It is represented in the 2021 Bundestag. The Left advocates disarmament and the expansion of the welfare state, calls for a billionaire and wealth tax and wants to reduce the burden on small and medium incomes.

Continued next page...

Table 8: (continued) The parties which participated in the German federal elections in 2009–2025

Party logo	Party description
20 	Die PARTEI, Partei für Arbeit, Rechtsstaat, Tierschutz, Elitenförderung und basisdemokratische Initiative (Party for Work, Rule-of-Law, Protection of Animals, Advancement of Elites and Grass-root Democratic Initiative) is founded in 2004 by members of the editorial team of the magazine "Titanic". As a satirical party, it parodies the established parties and the political system. It holds a mandate in the European Parliament and has been represented in the Bundestag by a deferred member of the SPD since 2020.
21 	DIE RECHTE (The Right), founded in 2012, is a right-wing extremist party with racially motivated utterances against foreigners and Islam. It calls for a ban on advertising in foreign languages and a wide range of changes in the asylum law.
22 	Die Violetten — für spirituelle Politik (The Violets — for spiritual policy) founded in 2001, claiming to represent 'alternative spiritual politics in the new age'.
23 	dieBasis, Basisdemokratische Partei Deutschland (Basic Democratic Party of Germany) was founded in 2020 in the context of protests against the government measures to combat the Covid-19 pandemic. Among other things, it speaks out against the digitization of the health system. DieBasis also calls for the commissioning of 'Nord Stream 2' and referendums at all levels.
24 	DKP, Deutsche Kommunistische Partei (German Communist Party) is founded in 1968 as the successor to the banned KPD. In the Marxist-Leninist tradition, it strives to establish a socialist system and to break with capitalist power and property relations. The Office for the Protection of the Constitution classifies it as left-wing extremist.
25 	DM, Deutsche Mitte (German Center), founded in 2013. It criticizes many policy areas and considers its own positions non-negotiable. It is against the euro and the EU, unlimited immigration, public broadcasting, compulsory schooling, and interest and taxes.
26 	du., Die Urbane. Eine HipHop Partei (The Urbans. A HipHop Party), founded in 2017. It proposes to transfer the key elements of hip-hop culture into political action. It is against discrimination and calls for the end of German arms exports and the withdrawal of Germany from NATO.
27 	DVU, Deutsche Volksunion (The German People's Union), founded as an association in 1971 and as a right-wing extremist political party in 1987. In 2011, it merged with the right-wing extremist National Democratic Party (NPD). The merged party was briefly called NPD–The People's Union, then again just NPD, and since 2023 it has called itself Die Heimat. The legality of the merger was disputed in court for a long time and was stopped by an interim injunction after several DVU state associations had filed a lawsuit against its formation.
28 	FAMILIE, Familien-Partei Deutschlands (The Family Party of Germany), founded in 1983, a party promoting family values.
29 	FDP, Freie Demokratische Partei (Free Democratic Party), founded in 1948, emphasizes the freedom and self-determination of the individual. Among other things, it calls for a reduction in taxes and duties as well as the reduction of bureaucratic regulations and relies on innovations to tackle climate change. The FDP was part of the federal government until November 2024.
30 	FREIE WÄHLER (Free Voters) was founded in 2009 as a nationwide association of local voter groups. They have been part of the Bavarian state government since 2018. The FREIE WÄHLER are calling for, among other things, a strengthening of local self-government, a simplification of tax law and a reduction in citizens' allowances.

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Table 8: (continued) The parties which participated in the German federal elections in 2009–2025

Party logo	Party description
31 	GRÜNE, BÜNDNIS 90/DIE GRÜNEN (Alliance 90/The Greens). The Greens were founded in 1980 and later joined forces with civil rights movements from the former GDR. They campaign for environmental protection, disarmament, renewable energies and gender equality, among other things. The Greens have been part of the federal government since 2021.
32 	III. Weg, DER DRITTE WEG (The Third way party) is founded in 2013. Anti-Semitism, racism, ethnic view of man and the striving for a social order based on historical National Socialism characterize the party. The Office for the Protection of the Constitution classifies it as right-wing extremist.
33 	LfK, Partei für Kinder, Jugendliche und Familien Lobbyisten für Kinder (Party for children, young people and families - lobbyists for children) is founded in 2021 and is committed to ensuring that the interests of minors and parents are given greater consideration in political decisions. It calls for more investment in education and families as well as a lowering of the voting age.
34 	LIEBE, Europäische Partei LIEBE (European party LOVE) is founded in 2018 and is a pro-European party. For the party, love is the starting point and driving force of all social coexistence and political action, towards fellow human beings, but also towards animals and nature.
35 	LKR, Liberal-Konservative Reformer (Liberal Conservative Reformers) is founded in 2015 by the former AfD federal spokesman Bernd Lucke. It represents economically liberal and conservative positions and calls for a fundamental reform of the EU. By converting the party from the AfD, it is represented by individual members of parliament in state parliaments and in the Bundestag.
36 	Menschliche Welt (The Human World) was founded in 2013. Its policies are based on a spiritual way of life and a decentralized common good economy. Issues such as peace policy, organic agriculture and economic justice play an important role in its program.
37 	MERA25 — Gemeinsam für Europäische Unabhängigkeit, (MERA25 — Together for European Independence) was founded first as Democracy in Europe (German: Demokratie in Europa) in 2020, and in 2021 was renamed to MERA25, in reference to the Greek party with the same name. The party is as part of the pan-European DiEM25. It calls for a radical change to a more directly democratic, more solidarity-based and more sustainable EU. Its goal is, among other things, the introduction of a "universal living income" and the socialization of key basic goods.
38 	MLPD, Marxistisch-Leninistische Partei Deutschlands (Marxist-Leninist Party of Germany) is a communist party founded in 1982. It refers to Marx, Engels, Lenin, Stalin and Mao Zedong and sees itself as a radical left-wing alternative to other political forces. It is classified as left-wing extremist by the Federal Office for the Protection of the Constitution.
39 	Nichtwähler, Partei der Nichtwähler (Party of Non-voters), founded in 1998, a party with a social democratic background, promoting improving representative democracy by introducing elements of direct democracy.
40 	NPD, National-demokratische Partei Deutschlands (National Democratic Party of Germany) founded in 1964 is a right-wing extremist party. It rejects free democracy and represents xenophobic and aggressive social-populist positions. The Federal Constitutional Court attests its political concept as disregarding the human dignity.

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Table 8: (continued) The parties which participated in the German federal elections in 2009–2025

Party logo	Party description
41 	ÖDP, Ökologisch-Demokratische Partei (Ecological Democratic Party) emerged from the ecology movement in 1981. Key issues include environmental and family policy, as well as democracy and the transparency of political processes. Since 2014, it has won one mandate in each of the European Parliament elections.
42 	PDV, Partei der Vernunft (Party of Reason), founded in 2009, promotes liberal ideas of the Austrian School of economic thought: minimal state, free market, decentralization of political power and subsidiarity.
43 	PdF, Partei des Fortschritts (Party of Progress) was founded in 2020. It calls for more direct democratic elements. It finds its own positions in a grassroots democratic process. Among other things, it calls for a consistent energy transition and the expansion of public transport. In 2024, it won a mandate in the European Parliament.
44 	PdH, Partei der Humanisten — Fakten, Freiheit, Fortschritt (Party of Humanists — Facts, Freedom, Progress). Founded in 2014, the PdH places people as self-determined, social and rational individuals at the centre of its politics. It opposes the connection between state and religion and wants to continuously adapt its positions to new scientific findings.
45 	PIRATEN, Piratenpartei Deutschland (Pirate Party of Germany) was founded in 2006 with a focus on internet policy issues such as data protection, digital copyright and net neutrality. They demand, among other things, a free and democratically controlled technical infrastructure and more powers for the federal government in education policy. From 2011 to 2017 they were represented in up to four state parliaments.
46 	pro Deutschland, Bürgerbewegung pro Deutschland (Pro-Germany Citizens' Movement), founded in 2005, a far-right populist party opposing illegal immigration and multi-national corporations and financial institutions.
47 	RENTNER Partei Deutschland (German Party of Pensioners) founded in 2002, a party of social welfare state bridging the interests of generations.
48 	REP, Die Republikaner (The Republicans), founded in 1983, a nationalist conservative party opposing immigration.
49 	SGP, Sozialistische Gleichheitspartei, Vierte Internationale (Socialist Equality Party, Fourth International), founded in 1971 as BSA, Bund Sozialistischer Arbeiter (Alliance of Socialist Workers) and called from 1997 to 2017 PSG, Partei für Soziale Gleichheit (Party of Social Equality), is a Trotskyist anticapitalist party. Its goals are the conquest of political power by the working class, the overthrow of capitalism and the "United Socialist States of Europe". The Federal Office for the Protection of the Constitution classifies it as left-wing extremist.
50 	SPD, Sozial-demokratische Partei Deutschlands (Social Democratic Party of Germany), founded in 1863. The SPD emerged from the workers' movement in 1875. The slogan of social justice is the starting point for many of its positions, for example in the party's labor, social and societal policies. Since the 2021 federal elections it the fourth time in its history that the party has a chancellor.

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Table 8: (continued) The parties which participated in the German federal elections in 2009–2025

Party logo	Party description
51  SSW <i>Für uns im Norden.</i>	SSW, Südschleswigscher Wählerverband (South Schleswig Association of Voters), was founded in 1948. It is the political lobby of the Danish minority and the Frisian ethnic group and is therefore exempt from the 5%-hurdle. Its focus is on northern Germany. Since 2021 the SSW is represented by one member of Bundestag.
52 	Team Todenhöfer, Die Gerechtigkeitspartei — Team Todenhöfer (The Justice Party — Todenhöfer's Team) was founded in 2020 by former CDU MP Jürgen Todenhöfer. It campaigns for an end to the Bundeswehr's foreign missions, is against national unilateral climate policy efforts and calls, among other things, for the construction of one million homes annually, a limitation of the term of office of MPs and a ban on party donations over 5,000 euros.
53 	Tierschutzallianz, Allianz für Menschenrechte, Tier- und Naturschutz (Alliance for Human Rights, Animal and Nature Conservation), founded in 2013. It calls for animal protection, animal experimentation-free research, more direct public participation, guaranteed basic income and better hygiene standards in hospitals.
54  Tierschutzpartei	Tierschutzpartei: Mensch Umwelt Tierschutz (Animal Protection Party: People–Environment–Animal Protection) was founded in 1993. It is particularly committed to protecting the environment and animals. One of its goals is to include basic rights for animals in the Basic Law with a separate article. Since 2014, it has won one mandate in each of the European Parliament elections.
55 	UNABHÄNGIGE, UNABHÄNGIGE für bürgernahe Demokratie (INDEPENDENTS for community-based democracy) is founded in 2002 and advocates more citizen participation and direct democracy. It calls for the introduction of referendums at federal level, advocates freedom of research and opinion and advocates transparent political processes.
56 	V-Partei3 — Partei für Veränderung, Vegetarier und Veganer (V-Party3 — Party for Change, Vegetarians and Vegans) is founded in 2016 and wants to draw attention to the effects of growth, consumption and eating behavior. It calls for a bio-vegan orientation in agriculture, the long-term withdrawal from livestock farming and the improvement of consumer, climate and animal protection.
57 	Volksabstimmung (Referendum party), founded in 1997, promotes direct democracy of the Swiss type.
58 	Volt, Volt Deutschland (Volt Germany) was founded in 2018, is part of a pan-European movement. Important goals include a fundamental reform of the EU, extensive digitalization of administration and the switch to renewable energies. It has been a member of the European Parliament since 2019 — currently with three mandates.
59 	WU, WerteUnion (The Values Union) is a German party founded in 2024 by transforming a seven-year-old registered association with the same name. The party calls for, among other things, a restrictive migration policy and the abolition of public broadcasting in its current form. It wants to reduce the influence of the parties in favor of direct democratic elements. According to its own information, the Values Union had around 4,000 members in 2022; with about 3,000 also being the members of the CDU.
60 	ZENTRUM, the party of Catholics and political Catholicism, was at its most important between 1871 and 1933, during the time of the German Empire and the Weimar Republic. Between 1917 and 1932, it provided the Chancellor of the Reich four times. After the Second World War, it was re-founded and has been active as a small party, especially in local politics with short representations in the Bundestag in 2022–2023, also in the European Parliament.

References

- [Armstrong II et al. 2014] Armstrong II DA, Bakker R, Carroll R, Hare Ch, Poole KT, Rosenthal H (2014) *Analyzing spatial models of choice and judgment with R*. CRC Press, Boca Raton FL
- [Arrow et al. 2002/2011] Arrow KJ, Sen AK, Suzumura K (eds) (2002/2011) *Handbook of social choice and welfare*, vols I and II. Elsevier, Oxford–Amsterdam
- [Ballester and Haeringer 2011] Ballester M, Haeringer G (2011) A characterization of the single-peaked domain. *Soc Choice Welf* 36(2): 305–322
- [Barberà 2011] Barberà S (2011) Strategy-proof social choice. In: Arrow KJ, Sen AK, Suzumura K (eds) *Handbook of social choice and welfare*, vol II. Elsevier, Oxford–Amsterdam: 731–832
- [Barberà and Moreno 2011] Barberà S, Moreno B (2011) Top monotonicity: A common root for single peakedness, single crossing and the median voter result. *Games and Economic Behavior* 73(2): 345–359
- [Barberà et al. 1993] Barberà S, Gul S, Stacchetti E (1993) Generalized median voter schemes and committees. *J Econ Th* 61(2): 262–289
- [Benoit and Laver 2012] Benoit K, Laver M (2012) The dimensionality of political space: Epistemological and methodological considerations. *European Union Politics* 13(2): 194–218. http://www.kenbenoit.net/pdfs/EUP-2012_Benoit-Laver.pdf. Cited 12 Apr 2019
- [Black 1948] Black D (1948) On the rationale of group decision-making. *J Polit Econ* 56(1): 23–34
- [Black 1958] Black D (1958) *The theory of committees and elections*. Cambridge University Press, Cambridge. 2nd ed. Kluwer Academic Publishers, Dordrecht, 1987
- [Blattberg 2009] Blattberg Ch (2009) Political philosophies and political ideologies. In: Blattberg Ch (ed) *Patriotic elaborations: Essays in practical philosophy*. McGill-Queen’s University Press, Montreal. First version in: *Public Affairs Quarterly* 15(3): 193–217 (July 2001)
- [Bobbio 1996] Bobbio N (1996) *Left and right: The significance of a political distinction*. Transl and introduction Cameron A. University of Chicago Press, Chicago
- [Bredereck et al. 2013] Bredereck R, Chen J, Woegingerz GJ (2013) Are there any nicely structured preference profiles nearby? *Proc 23rd Int Joint Conf on Artificial Intelligence*, Beijing, 3–9 August, 2013. AAAI Press, Palo Alto CA: 62–68. <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.377.3567&rep=rep1&type=pdf>. Cited 13 Aug 2019
- [Budge and McDonald 2007] Budge I, McDonald MD (2007) Election and party system effects on policy representation: Bringing time into a comparative perspective. *Electoral Studies* 26(1): 168–179
- [Budge et al. 2001] Budge I, Klingemann HD, Volkens A, Bara J, Tanenbaum E (2001) *Mapping policy preferences: Estimates for parties, electors and governments 1945–1998*. Oxford University Press, Oxford
- [BüSo 2021] BüSo (2021) Wikipedia (EN). Cited 02 Dec 2021.
- [Bundeswahlleiterin 2025] Bundeswahlleiterin (2025) Wahl zum 21. Deutschen Bundestag am 23. Februar 2025. Endgültige Ergebnisse nach Wahlkreisen. [Election of the 21st German Bundestag on February 23, 2025. Vol 3: Final results by constituencies]. Bundeswahlleiter, Wiesbaden. https://bundeswahlleiterin.de/info/presse/mitteilungen/bundestagswahl-2025/29_25_endgueltiges-ergebnis.html <https://bundeswahlleiterin.de/bundestagswahlen/2025/ergebnisse/bund-99.html>. Cited 22 Mar 2025

- [Bundeszentrale für politische Bildung 2025] Bundeszentrale für politische Bildung (2025). Wahl-O-Mat. <https://www.bpb.de/themen/wahl-o-mat/>. Cited 19 Feb 2025
- [Carlisle 2005] Carlisle RP (2005) Encyclopedia of politics: The left and the right, vol 2. Sage Publications, Thousand Oaks MI
- [Carroll et al. 2013] Carroll R, Lewis JB, Lo J, Poole KT, Rosenthal H (2013) The structure of utility in spatial models of voting. *Am J Pol Sci* 57(4): 1008–1028. https://scholar.princeton.edu/sites/default/files/jameslo/files/ajps_utility.pdf. Cited 12 Apr 2019
- [Condorcet 1785] Condorcet MJAN (1785) Essai sur l’application de l’analyse à la probabilité des décisions rendues à la pluralité des voix [Essay on the application of probabilistic analysis to decisions made by plurality vote]. Imprimerie Royale, Paris
- [Condorcet 1994] Condorcet (1994) Foundations of social choice and political theory. Transl McLean I and Hewitt F. Edgar Elgar, Cheltenham
- [Conitzer 2009] Conitzer V (2009) Eliciting single-peaked preferences using comparison queries. *J Artificial Intelligence Research* 35(June): 161–191. <https://users.cs.duke.edu/~conitzer/singlepeakedJAIR.pdf>. Cited 30 Aug 2019
- [Cornaz et al. 2013] Cornaz D, Galand L, Spanjaard O (2013) Kemeny elections with bounded single-peaked or single-crossing width. *Proc 23rd Int Joint Conf on Artificial Intelligence*, Beijing, 3–9 August, 2013. AAAI Press, Palo Alto CA: 76–82. <https://www.ijcai.org/Proceedings/13/Papers/022.pdf>. Cited 30 Aug 2019
- [Downs 1957] Downs A (1957) An economic theory of democracy. Harper and Row, New York
- [Durand 2015] Durand C (2015) Polls on national independence: The Scottish case in a comparative perspective. Paper at the conference of the European Survey Research Association (ESRA), Reykjavik, 13–17 July, 2015. See also Durand C (19.09.2014) Ah! les sondages. <http://ahlessondages.blogspot.ca/>. Cited 12 Aug 2019
- [Elkind et al. 2014] Elkind E, Faliszewski P, Skowron P (2014) A characterization of the single-peaked single-crossing domain. *Proc 28th Conf of Association for Advancement of Artificial Intelligence*, Québec, July 27–31, 2014. AAAI Press, Palo Alto CA: 654–660. <https://pdfs.semanticscholar.org/aa32/69c207ce8da5f1883947d6fb5a31863d0e8d.pdf>. Cited 12 Aug 2019
- [Enelow 1994] Enelow JM (1994) A test of the predictive dimensions model in spatial voting theory. *Public Choice*, 78(2): 155–170. <http://www.la.utexas.edu/hinich/files/Politics/A%20Test%20of%20the%20Predictive%20Dimensions%20Model.pdf>. Cited 30 Aug 2019
- [Enelow and Hinich 1984] Enelow JM, Hinich MJ (1984) The spatial theory of voting: an introduction. Cambridge University Press, New York
- [Enelow and Hinich 1990] Enelow JM, Hinich MJ (eds) (1990) Advances in the spatial theory of voting. Cambridge University Press, New York
- [Escoffier et al. 2008] Escoffier B, Lang J, Öztürk M (2008) Single-peaked consistency and its complexity. *Proc 18th Eur Conf on Artificial Intelligence (ECAI-08)*, Patras, Greece: 366–370. <https://www.lamsade.dauphine.fr/~lang/papers/elo-ecai08.pdf>. Cited 12 Aug 2019
- [Eysenck 1955] Eysenck H (1955) The psychology of politics. Frederick A Praeger, New York. Republished by Routledge, 1998.
- [Faye 1996] Faye JP (1996) Le siècle des idéologies [The century of ideologies]. Armand Colin, Paris. 2nd ed Pocket, Paris, 2002

- [Ferguson 1941] Ferguson LW (1941) The stability of the primary social attitudes: I. Religionism and humanitarianism. *Journal of Psychology*, 12(2): 283–288.
- [Friendly 2002] Friendly M (2002) Corrgrams: Exploratory displays for correlation matrices. *American Statistician* 56(4): 316–324. <http://www.datavis.ca/papers/corrgram.pdf>. Cited 12 Aug 2019
- [Friendly and Kwan 2003] Friendly M, Kwan E (2003) Effect ordering for data display. *Computational Stat Data Analysis* 43(4): 509–539
- [Gaertner 2001] Gaertner W (2001) Domain conditions in social choice theory. Cambridge University Press, Cambridge
- [Garzia and Marschall 2014] Garzia D, Marschall S (eds) (2014) Matching voters with parties and candidates: Voting advice applications in a comparative perspective. ECPR Press, Colchester UK
- [Gauchet 1996] Gauchet M (1996) Right and left. In Nora P, Kritzman LD (eds) *Realms of memory: Rethinking the French past*, vol 1—Conflicts and divisions. Transl Goldhammer A. Columbia University Press, New York: 240–298
- [Gehrlein 2002] Gehrlein WV (2002) Condorcet’s paradox and the likelihood of its occurrence: Different perspectives on balanced preferences. *Theor Decis* 52(2): 171–199
- [Gehrlein and Lepelley 2011] Gehrlein WV, Lepelley D (2011) Voting paradoxes and group coherence. Springer, Heidelberg
- [Gemenis 2013] Gemenis K (2013) Estimating parties’ policy positions through voting advice applications: some methodological considerations. *Acta Politica* 48: 268–295. <https://link.springer.com/article/10.1057%2Fap.2012.36>. Cited 13 Aug 2019
- [Germann and Mendez 2016] Germann M, Mendez F (2016) Dynamic scale validation reloaded. Assessing the psychometric properties of latent measures of ideology in VAA spatial maps. *Quality and Quantity* 50(3): 981–1007
- [Germann et al. 2015] Germann M, Mendez F, Wheatley J, Serdült U (2015) Spatial maps in voting advice applications: the case for dynamic scale validation. *Acta Politica* 50: 214–238
- [Giddens 1994] Giddens A (1994) Beyond left and right, the future of radical politics. Stanford University Press, Stanford CA
- [Gill and Hangartner 2010] Gill J, Hangartner D (2010) Circular data in political science and how to handle it. *Political Analysis* 18(3): 316–336
- [Gosse 2005] Gosse VE (2005) Rethinking the new left: An interpretative history. Palgrave Macmillan, New York
- [Grandmont 1978] Grandmont JM (1978) Intermediate preferences and the majority rule. *Econometrica* 46(2): 317–330
- [Grofman 1985] Grofman B (1985) The neglected role of the status quo in models of issue voting. *J of Politics* 47(1): 230–237
- [Grofman and Feld 1988] Grofman B, Feld S (1988) Rousseau’s general will: A Condorcetian perspective. *Am Polit Sci Rev* 82(2): 567–578
- [Heywood 2017] Heywood A (2017) Political ideologies: an introduction. 6th ed. Macmillan, Basingstoke.

- [Hinich and Munger 1994] Hinich MJ, Munger MC (1994) *Ideology and the theory of political choice*. University of Michigan Press, Ann Arbor MI
- [Hinich and Munger 1997] Hinich MJ, Munger MC (1997) *Analytical politics*. Cambridge University Press, New York
- [Husson et al. 2011] Husson F, Lê S, Pagès J (2011) *Exploratory multivariate analysis by example using R*. CRC Press, Boca Raton FL
- [Jackson 1988] Jackson JE (1988) *A user's guide to principal components*. Wiley, New York
- [Jeong et al. 2009] Jeong DH, Ziemkiewicz C, Ribarsky W, Chang R (2009) Understanding principal component analysis using a visual analytics tool. VACCINE publication 235 (Visual Analytics for Command, Control, and Interoperability Environments). Purdue University, West Lafayette IN. <http://www.purdue.edu/discoverypark/vaccine/publications.php>. Cited 12 Apr 2019
- [Kieskompas 2006] Kieskompas [Election compass] (2006). <http://www.kieskompas.nl/>. Cited 13 Aug 2019
- [Klingemann et al. 2006] Klingemann HD, Volkens A, Bara JL, Budge J, McDonald MD (eds) (2006) *Mapping policy preferences II: Estimates for parties, electors, and governments in Eastern Europe, European Union, and OECD 1990–2003*. Oxford University Press, Oxford
- [Knapp and Wright 2006] Knapp A, Wright V (2006) *The government and politics of France*, 5th ed. Routledge, Oxford
- [Kriesi 2008] Kriesi H (2008) Political mobilisation, political participation and the power of the vote. *West European Politics* 31(1/2): 147–168
- [Kriesi et al. 2006] Kriesi H, Grande E, Lachat R, Dolezal M, Bornschie S, Frey T (2006) Globalization and the transformation of the national political space: Six European countries compared. *European J of Political Research* 45(6): 921–956. <http://doi.org/10.1111/j.1475-6765.2006.00644.x>. Cited 12 Apr 2019
- [Krzanowski 1988] Krzanowski WJ (1988) *Principles of multivariate analysis*. Clarendon Press, Oxford
- [Linhart and Shikano 2007] Linhart E, Shikano S (2007) Die Generierung von Parteipositionen aus vorverschluselten Wahlprogrammen für die Bundesrepublik Deutschland [The generation of party positions from encoded election programs for the Federal Republic of Germany]. MZES working paper. Mannheim, Mannheimer Zentrum für Europäische Sozialforschung. <http://www.mzes.uni-mannheim.de/publications/wp/wp-98.pdf>. Cited 12 Aug 2019
- [Lipset 1960] Lipset SM (1960) *Political man: The social bases of politics*. Doubleday, Garden City NY
- [List et al. 2013] List C, Luskin R, Fishkin J, McLean I (2013) Deliberation, single-peakedness, and the possibility of meaningful democracy: evidence from deliberative polls. *J of Politics* 75(1): 80–95 [http://eprints.lse.ac.uk/46863/1/Deliberation,%20single-peakedness,%20and%20the%20possibility%20of%20meaningful%20democracy\(lsero\).pdf](http://eprints.lse.ac.uk/46863/1/Deliberation,%20single-peakedness,%20and%20the%20possibility%20of%20meaningful%20democracy(lsero).pdf). Cited 13 Aug 2019
- [Little 1988] Little RJA (1988) A test of missing completely at random for multivariate data with missing values. *J Amer Stat Ass* 83(404): 1198–1202
- [Little and Rubin 2002] Little RJA, Rubin DB (2002) *Statistical analysis with missing data*, 2nd ed. Wiley, New York

- [Luther 2012] Luther KR (2012) Peter Mair and the Europeanization of parties and party systems. Keele European parties research unit (KEPRU) working paper 37. <https://www.keele.ac.uk/media/keeleuniversity/fachumsocsci/spire/politics/Peter%20Mair%20and%20Europeanization%20of%20parties%20and%20party%20systems.pdf>. Cited 12 Aug 2019
- [Mahoney et al. 1984] Mahoney J, Coogole CL, Banks PD (1984) Values in presidential inaugural addresses: A test of Rokeach's two-factor theory of political ideology. *Psychological Reports* 55(3): 683–686
- [Mair 2007] Mair P (2007) Political parties and party systems. In: Graziano P, Vink MP (eds) *Europeanization: New research agendas*. Palgrave Macmillan, Basingstoke: 154–166
- [Manin 1997] Manin B (1997) *The principles of representative government*. Cambridge University Press, Cambridge
- [Marx 1867] Marx K (1867) *Das Kapital*. Band I. Verlag von Otto Meisner, Hamburg
- [Matthews 1979] Matthews SA (1979) A simple direction model of electoral competition. *Public Choice* 34(2): 141–156
- [McLean and McMillan 2009] McLean I, McMillan A (2009) Right-wing. In: McLean I, McMillan A (eds) *The concise Oxford dictionary of politics*, 3rd ed. Oxford University Press, Oxford
- [Mendez and Wheatley 2014] Mendez F, Wheatley J (2014) Using VAA-generated data for mapping partisan supporters in the ideological space. In: Garzia D, Marschall S (eds) *Matching voters with parties and candidates: Voting advice applications in a comparative perspective*. ECPR Press, Colchester UK: 161–174
- [Mirrlees 1971] Mirrlees J (1971) An exploration in the theory of optimal income taxation. *Rev Econ Stud* 38(2): 175–208. <http://people.tamu.edu/~ganli/publicecon/mirrlees71.pdf>. Cited 30 Aug 2019
- [Mitchell 2007] Mitchell BP (2007) *Eight ways to run the country: A new and revealing look at left and right*. Praeger, Westport CN
- [Moulin 1988] Moulin H (1988) *Axioms of cooperative decision making*. Cambridge–New York, Cambridge University Press
- [Müller-Rommel and Bértoa 2016] Müller-Rommel F, Bértoa FC (eds) (2016) *Party politics and democracy in Europe. Essays in honour of Peter Mair*. Routledge, London–New York
- [Neundorf 2009] Neundorf A (2009) Growing up on different sides of the Wall — a quasi-experimental test: Applying the left-right dimension to the German mass public. *German Politics* 18(2): 201–225
- [Neundorf 2011] Neundorf A (2011) Die Links-Rechts-Dimension auf dem Prüfstand: Ideologisches Wählen in Ost- und Westdeutschland 1990–2008 [Testing the left-right dimension: Ideological voting in East and West Germany, 1990–2008]. *Politische Vierteljahresschrift* 45 (special issue 'Voting in Germany'): 227–250
- [Nurmi 1999] Nurmi H (1999) *Voting paradoxes and how to deal with them*. Springer, Berlin
- [Otjes and Louwerse 2014] Otjes S, Louwerse T (2014) Spatial models in voting advice applications. *Electoral Studies* 36: 263–271.
- [Pearson 1901] Pearson K (1901) On lines and planes of closest fit to systems of points in space. *Philosophical Magazine* 2(11): 559–572.

- [Peters and Lackner 2020] Peters D, Lackner M (2020) Preferences single-peaked on a circle. *Journal of Artificial Intelligence Research*, 68: 463–502. Short version under the same title in: *Proceedings of the 31st AAAI Conference on Artificial Intelligence 2017 (AAAI-17)*: 649–655
- [Poole 2005] Poole KT (2005) *Spatial models of parliamentary voting*. Cambridge University Press, New York
- [Poole and Rosenthal 2007] Poole KT, Rosenthal H (2007) *Ideology and congress*. Transaction Press, Piscataway NJ
- [Politicalresearch.org 2021] Politicalresearch.org (2021) Repression and ideology: The legacy of discredited centrist/extremist theory. <https://www.politicalresearch.org/repression-and-ideology-legacy-discredited-centristextremist-theory>. Cited 17 Nov 2021
- [Puppe 2018] Puppe C (2018) The single-peaked domain revisited: A simple global characterization. *J Econ Th*, 176(C): 55–80
- [Rabinowitz and Macdonald 1989] Rabinowitz G, Macdonald SE (1989) A directional theory of issue voting. *Am Pol Sc Rev* 83(1): 93–121
- [Rokeach 1973] Rokeach M (1973) *The nature of human values*. Free Press.
- [Regenwetter et al. 2006] Regenwetter M, Grofman B, Marley AAJ, Tsetlin I (2006) *Behavioral social choice: Probabilistic models, statistical inference, and applications*. Cambridge University Press, Cambridge
- [Roberts 1977] Roberts KWS (1977) Voting over income tax schedules. *J Public Econ* 8(3): 329–340
- [Rous and Lee 1978] Rous GL, Lee DE (1978) Freedom and equality: Two values of political orientation. *J Communication* 28(1): 45–51
- [Ruyppers et al. 2005] Ruyppers J, Austin M, Carter P, Murphy TG (2005) *Canadian and world politics*. Emond Montgomery Publications Limited, Toronto
- [Saari 1994] Saari D (1994) *Geometry of voting*. Springer, Heidelberg–Berlin
- [Saari 1995] Saari D (1995) *Basic geometry of voting*. Springer, Heidelberg–Berlin
- [Saporiti 2009] Saporiti A (2009) Strategy-proofness and single-crossing. *Theoretical Economics* 4(2): 127–163
- [Saporiti and Tohmé 2006] Saporiti A, Tohmé F (2006) Single-crossing, strategic voting and the median choice rule. *Soc Choice and Welf* 26(2): 363–383
- [Schofield 1985] Schofield NJ (1985) *Social choice and democracy*. Springer, New York
- [Schumpeter 1942] Schumpeter J (1942) *Capitalism, socialism and democracy*. Introduction Swedberg R. Routledge, London–New York, 1994. <https://eet.pixel-online.org/files/etranslation/original/Schumpeter,%20Capitalism,%20Socialism%20and%20Democracy.pdf>. Cited 1 Sep 2019
- [Seber 1984] Seber GAF (1984) *Multivariate observations*. Wiley, New York
- [Skowron et al. 2013] Skowron P, Yu L, Faliszewski P, Elkind E (2013) The complexity of fully proportional representation for single-crossing electorates. *Proc 6th Int Symposium on Algorithmic Game Theory*, Aachen, October 21–23. Springer LNCS 8146, Berlin–Heidelberg: 1–12. <https://www.mimuw.edu.pl/~ps219737/singleCrossing.pdf>. Cited 13 Aug 2019
- [Smithies 1941] Smithies A (1941) Optimum location in spatial competition. *J Polit Econ* 49: 423–39

- [Sui et al. 2013] Sui X, Francois-Nienaber A, Boutilier C (2013) Multi-dimensional single-peaked consistency and its approximations. Proc 23 Int Joint Conference on Artificial Intelligence, Beijing, August 3–9, 2013. AAAI Press, Palo Alto CA: 375–382. <https://www.ijcai.org/Proceedings/13/Papers/063.pdf>. Cited 13 Aug 2019
- [Sulakshin 2010] Sulakshin S (2010) A quantitative political spectrum and forecasting of social evolution. *Int J Interdisciplinary Soc Sc* 5(4): 55–66
- [Tangian 2011] Tangian A (2011) Flexicurity and political philosophy. Nova Publishers, New York
- [Tangian 2014] Tangian A (2014) Mathematical theory of democracy. Springer, Berlin–Heidelberg. <https://link.springer.com/book/10.1007/978-3-642-38724-1>. Cited 3 Jun 2025.
- [Tangian 2020] Tangian A (2020) Analytical theory of democracy. Vols 1 and 2. Springer, Cham (CH)
- [Tangian 2022a] Tangian A (2022a) Analysis of the 2021 German federal elections. 1/4. Representativeness of the Parties and the Bundestag. ECON Working paper 151 (January 2022). Karlsruhe Institute of Technology, Karlsruhe. <https://publikationen.bibliothek.kit.edu/1000143156>. Cited 3 Jun 2025.
- [Tangian 2022b] Tangian A (2022b) Analysis of the 2021 Bundestag elections. 2/4. Political spectrum. ECON Working paper 152 (January 2022). Karlsruhe Institute of Technology, Karlsruhe. <https://publikationen.bibliothek.kit.edu/1000143157>. Cited 3 Jun 2025.
- [Tangian 2022c] Tangian A (2022c) Analysis of the 2021 Bundestag elections. 2/4. Tackling the Bundestag growth. ECON Working paper 153 (January 2022). Karlsruhe Institute of Technology, Karlsruhe. <https://publikationen.bibliothek.kit.edu/1000143158>. Cited 3 Jun 2025.
- [Tangian 2022d] Tangian A (2022d) Analysis of the 2021 Bundestag elections. 3/4. The third vote application. ECON Working paper 154 (January 2022). Karlsruhe Institute of Technology, Karlsruhe. <https://publikationen.bibliothek.kit.edu/1000143159>. Cited 3 Jun 2025.
- [Tangian 2025a] Tangian A (2025a) Analysis of the 2025 Bundestag elections. 1/4. Imperfection of the Electoral Reform. ECON Working Paper 167 (June 2025). Karlsruhe Institute of Technology, Karlsruhe. <https://econpapers.wiwi.kit.edu>
- [Tangian 2025b] Tangian A (2025b) Analysis of the 2025 Bundestag Elections. 2/4. Representativeness of the Parties and the Bundestag. ECON Working Paper 168 (June 2025). Karlsruhe Institute of Technology, Karlsruhe. <https://econpapers.wiwi.kit.edu>.
- [Tangian 2025c] Tangian A (2025c) Analysis of the 2025 Bundestag Elections. 3/4. The Third Vote Perspective. ECON Working Paper 169 (June 2025). Karlsruhe Institute of Technology, Karlsruhe. <https://econpapers.wiwi.kit.edu>.
- [Taylor 2006] Taylor J (2006) Where did the party go: William Jennings Bryan, Hubert Humphrey, and the Jeffersonian legacy. University of Missouri Press, Columbia MO
- [Van Houweling and Sniderman 2005] Van Houweling RP, Sniderman PM (2005) The political logic of a Downsian space. Institute of Governmental Studies UC Berkeley. escholarship.org/uc/item/3858b03t. Cited 12 Apr 2019
- [Voda 2014] Voda P (2014) Class voting in West and East. Paper at the 8th ECPR General Conference, University of Glasgow, September 3–6, 2014. <https://ecpr.eu/Filestore/PaperProposal/3626ab70-3ab7-4361-9421-bc8321828daa.pdf>. Cited Aug 11 2019
- [Volkens et al. 2013] Volkens A, Bara J, Budge I, McDonald MD, Klingemann HD (eds) (2013) Mapping policy preferences from texts: Statistical solutions for manifesto analysts. Oxford University Press, Oxford

- [Vote Match Europe 2019] Vote Match Europe (2019). <http://www.votematch.eu/>. Cited 4 Sep 2019
- [Wagner and Ruusuvirta 2012] Wagner M, Ruusuvirta O (2012) Matching voters to parties: Voting advice applications and models of party choice. *Acta Politica* 47(4): 400–422
- [Ware 1996] Ware A (1996) *Political parties and party systems*. Oxford University Press, Oxford
- [Weber 1921] Weber M (1921) *Economy and society*, ed Roth G and Wittich C, 2 vols. University of California Press, Berkeley and Los Angeles, 1978
- [Wheatley 2012] Wheatley J (2012) Using VAAs to explore the dimensionality of the policy space: Experiments from Brazil, Peru, Scotland and Cyprus. *Int. J. Electronic Governance* 5(3/4): 318–348. http://www.zora.uzh.ch/95094/1/Wheatley_Int_J_Electronic_Governance_2012.pdf. Cited 12 Apr 2019
- [Wheatley 2015] Wheatley J (2015) Cleavage structures and ideological dimensions in English politics: some evidence from VAA data. Paper for the ECPR General Conference, Montreal, 26–29 August, 2015. <https://ecpr.eu/Filestore/PaperProposal/7ed135a0-a554-435c-97a9-715e2882a6c4.pdf>. Cited 12 Apr 2019
- [Wheatley et al. 2014] Wheatley J, Carman C, Mendez F, Mitchell J (2014) The dimensionality of the Scottish political space: Results from an experiment on the 2011 Holyrood elections. *Party Politics* 20(6): 864–878
- [Wilson 2004] Wilson MS (2004) Values and political ideology: Rokeach’s two-value model in a proportional representation environment. *New Zealand J Psychology* 33(3): 155–162
- [WZB 2019] WZB (2019) The Manifesto project. WZB, Berlin. <https://www.wzb.eu/en/research/dynamics-of-political-systems/democracy-and-democratization/projects/the-manifesto-project>. Cited 13 Aug 2019
- [You vote EU 2019] You vote EU (2019). <https://yourvotematters.eu/>. Cited 3 Sep 2019
- [Young 1988] Young HP (1988) Condorcet’s theory of voting. *Am Polit Sci Rev* 82(4): 1231–1244
- [Züll and Scholz 2015] Züll C, Scholz E (2015) Who is willing to answer open-ended questions on the meaning of left and right? *Bulletin of Sociological Methodology* 127(1): 26–42

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