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by Andranik S. Tangian

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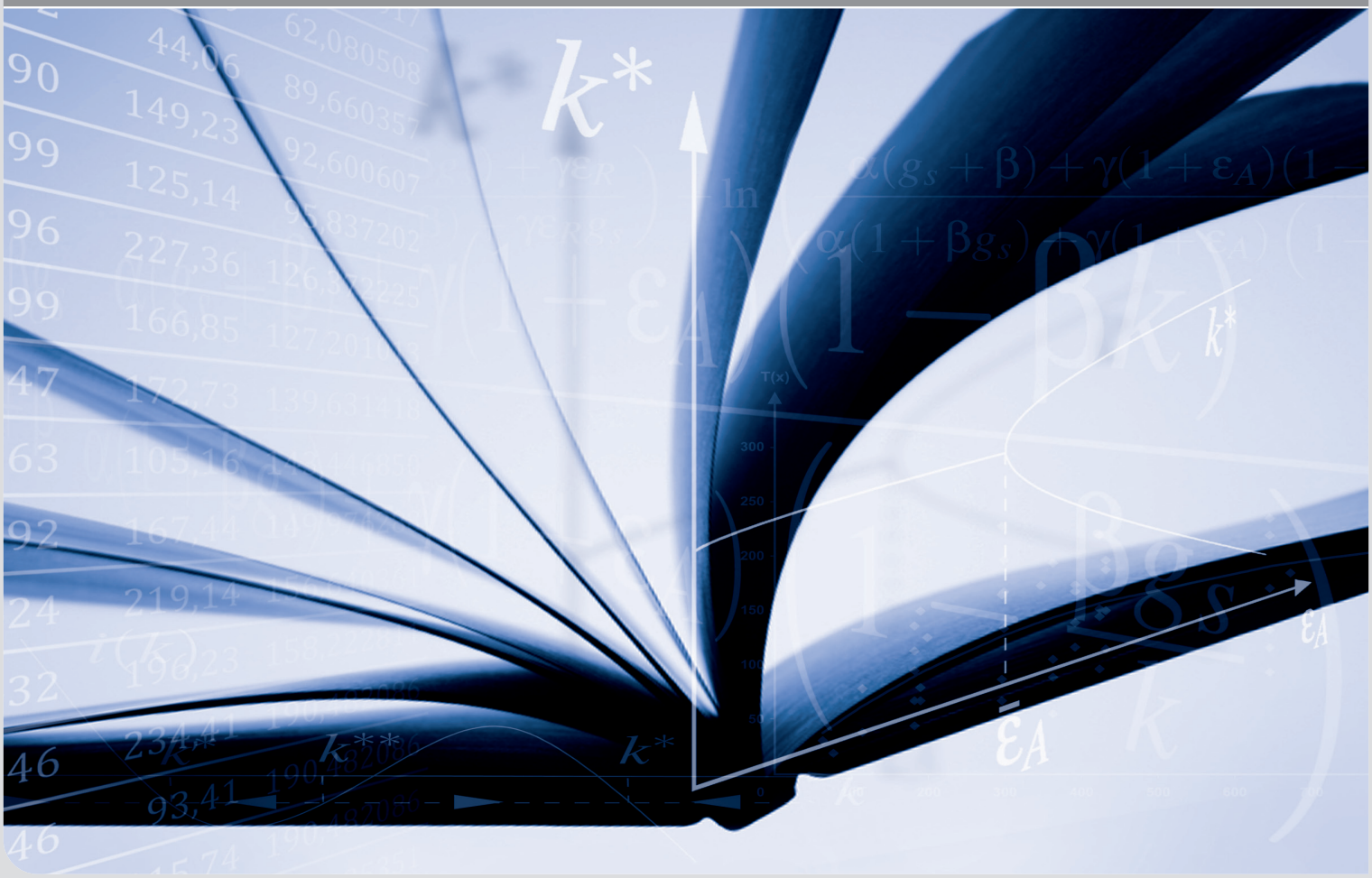

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Abstract

This is the last of four papers devoted to the 2021 German federal elections continuing our analysis of the 2009, 2013 and 2017 Bundestag elections. It is shown that the policy representation by the Bundestag could be improved using the alternative Third Vote election method. Under the Third Vote, electors cast no votes for parties by name. The electoral ballot consists of questions on topical policy issues ('General speed limit on motorways?'—Yes/No, 'Germany should increase its defense spending?'—Yes/No, etc.), and the parties answer to these questions before the elections — as required by the *Wahl-O-Mat*, the German voting advice applications. However, the Third Vote is not concerned with individual advices or individual voting intermediation. The electoral ballots are processed to construct the electorate's policy profile with balances of public opinion on all the issues. Then the matching of the parties' profiles with the electorate's profile is measured using the parties' indices of popularity (average percentage of electors represented on all the issues) and universality (percentage of questions when a majority is represented). These indices of representativeness are used instead of the conventional index 'number of votes received' to define the party quotas in the Bundestag. This method is hypothetically applied to reallocate the Bundestag seats to the eligible parties, resulting in a considerable gain in the Bundestag's representativeness. Finally, we discuss mixed election procedures combining the Third Vote with the conventional voting by party name and analyze possible implications.

Keywords: Representative democracy, elections, theory of voting, proportional representation.

JEL Classification: D71

Contents

1 Introduction

This is the last of four papers devoted to the 2021 German federal elections continuing our analysis of the Bundestag elections in 2009, 2013 and 2017 [Tangian 2014, 2017, 2020] by the methods of the mathematical theory of democracy. We show how policy representation by the Bundestag could be improved using the alternative Third Vote election method. The structure of the paper follows selected sections of [Tangian 2020, Chapter 10], from which we quote (with no special reference) for the reader's convenience.

Under representative democracy, participation by the people — the main purpose of democracy — is realized through the election of representatives. The representatives are empowered, on behalf of the people, to make political decisions and supervise vital attributes of democracy such as the rule of law, human rights, and freedom of the press. Therefore, representative democracy is only truly *democratic* if the elected actually *represent* the public interest. Correspondingly, the main criterion of democratic performance is the degree of representativeness of the governing institutions.

In [Tangian 2020, Section 5.3], two major historical concepts of political representation are discussed: one that is descriptive, requiring representatives to be local delegates and leading to proportional representation, and an agent one, allowing representatives to be external trustees and leading to majoritarianism. These concepts are implemented in different electoral systems and different methods of allocating parliamentary seats. The German electoral system with its two simultaneous votes combines both: the first 299 Bundestag seats are given to 299 deputies from 299 constituencies elected locally by the first vote (according to the descriptive concept), and the next 299 seats are distributed among eligible parties to adjust their factions to the party quotas in proportions of the second votes cast nationwide (according to the agent concept).

At the same time, the fundamental question, discussed since Rousseau's *Social Contract*, remains open: how, exactly, is political representation related to government by the people, or democracy? The related problems were realized shortly after World War II, when the concept of *policy representation* was introduced to monitor the quality of political intermediation; for a brief survey see [Tangian 2017]. The concept of policy representation complements the descriptive and agent concepts in the following way: the descriptive concept is concerned with the question *'Who is represented?'*; the agent concept with *'Who are representatives?'*; but both disregard the question *'What is represented?'* — which is the subject of policy representation.

It is not surprising that policy representation is missing from the debates of the 18th century, when the idea of representation was coined. The acute questions of that time were voting rights, property qualifications, gender restrictions, assembly composition, etc., that is, who should vote (who is represented), and eligibility conditions for public offices, that is, who can be elected and to which offices (who are representatives). The uneducated common folk had no policy preferences on most policy issues (with a few exceptions for voting rights, equality before the law or taxation); the mass-communication possibilities were limited, and politicians seldom, if ever, made their positions on every issue public. Since political decisions were entrusted to elected representatives, government by the people was not on the agenda; furthermore, the Founding Fathers of the United States were explicitly against democracy [Manin 1997, Ch. 3]. Correspondingly, neither the American Constitution nor any Amendment to it ever refers to democracy; democracy is also completely missing from the French post-revolutionary constitutions; see [Tangian 2020, Section 5.7].

Now the situation is different. The voting rights and eligibility for offices are no longer disputed, because all restrictions for citizens are removed. The population has become more advanced politically, mass media make information available, and politicians' manifestos are publicly discussed. Policy representation is becoming more important as a bridge between representative government and government by the people. However, candidates' positions and party manifestos receive little attention from the electorate. The question *'What is represented?'* is still insufficiently elucidated, and the existing electoral systems bear

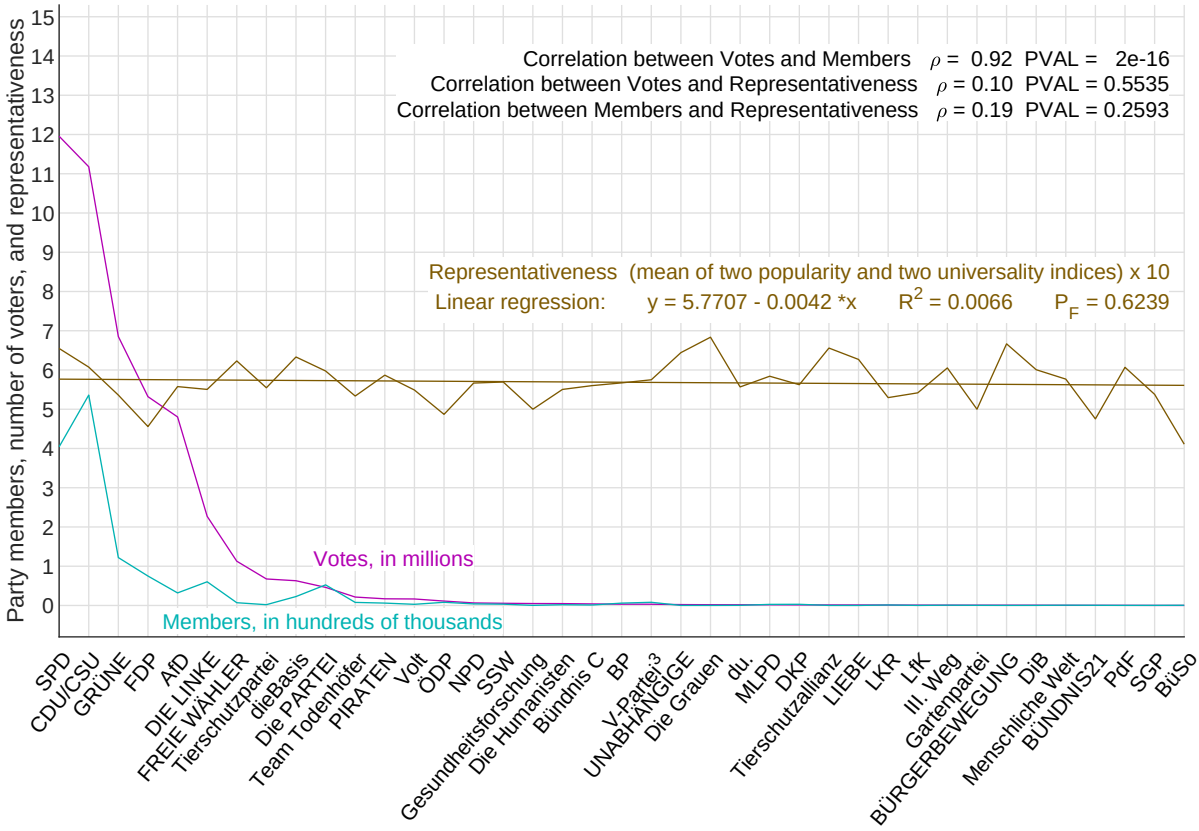

Figure 1: Size of German parties at the 2021 Bundestag election, votes received, and the mean representativeness index

some of the blame for that.

Figure 1 illustrates the coordination between the three aspects (concepts) of representation. The question *'Who is represented?'* is depicted by the curve 'Votes', because the represented are voter groups who cast their votes for this or that party. The question *'Who are representatives?'* is visualized by the curve 'Members', for the size of the parties that represent the corresponding voter groups, i.e. the representatives are parties. The question *'What is represented?'* is associated with the curve 'Representativeness', because the matters to be represented are the people's policy preferences, whose representation quality is characterized by the mean of the parties' popularity and universality indices defined and computed in [Tangian 2022a].

As seen in Figure 1, the first two aspects of representation, 'Votes' and 'Members', which were carefully deliberated by the founders of representative government, are in a perfect agreement with each other — the larger the voter group, the larger its party-representative. This follows from the high correlation of 0.92 between the curves 'Votes' and 'Members' with the statistical significance of order 10^{-16}. The causality here is likely two-way. On the one hand, large parties with proven leadership enjoy established reputations, have large networks, raise considerable funds for electoral campaigns and are constantly presenting themselves in the media. On the other hand, the electoral success itself attracts members through political inclusion, participation in policy making and by enhancing personal influence and career prospects.

At the same time, Figure 1 demonstrates no good coordination between the first two aspects of representation and the third one, policy representation, which was not foreseen in the initial design of representative government. This is visualized by the irregular behavior of the curve 'Representativeness', as opposed to that of the curves 'Votes' and 'Members'. As indicated at the top of the figure, the correlation of

'Representativeness' with 'Votes' and 'Members' is statistically insignificant,[1] meaning that the mutual understanding even between large parties and their numerous adherents is not extendable to the whole of the population. Indeed, good coordination between the first two aspects of representation is merely inherent in the near-party circles. The questions *'Who is represented?'* and *'Who are representatives?'*, which single out the groups of party adherents and party members, are not directly addressed to all the people. The 38 parties considered here (the union of CDU and CSU is regarded as one party) express primarily the policy preferences of ca. 1.23 million party members [Niedermayer 2018, p. 6] and not necessarily that of the 61.2 million of eligible German voters [Bundeswahlleiter 2021, p. 9].

We have here what [Dahl 1989] calls *polyarchy* — rule by the many, but obviously not by the people: even large parties that win elections can fail on representing public opinion. This is generally inherent in one-party systems, and, as we see, this is also true for Germany.

Thus, the answer to the question 'What is most important for a party's electoral success?' can only be: the party's size. This empirical evidence is not what one would expect from representative democracy, indicating a serious gap in its implementation. It seems that elections with voting by party name are designed for two historical concepts of representation — descriptive and agent — but not for policy representation. Indeed, the existing election methods divert the voters' attention away from specific policies, focusing on personalities and political labels.

Since the two historical types of representation are backed up by their own election methods, it is natural to enhance the third type of representation, policy representation, using, for example, the *Third Vote* election method — a dedicated election procedure introduced in [Tangian 2014, Tangian 2017] and studied both theoretically and empirically in [Tangian 2020]. Under the Third Vote, electors cast no votes for parties by name. The electoral ballot consists of questions on topical policy issues ('General speed limit on motorways?'—Yes/No, 'Germany should increase its defense spending?'—Yes/No, etc.), and the parties answer to these questions before the elections as required by the *Wahl-O-Mat*, the German voting advice applications. However, the Third Vote is not concerned with individual advices or individual voting intermediation. The electoral ballots are processed to construct the electorate' policy profile with the balances of public opinion on every issue. Then the matching of the parties' profiles with the electorate's profile is measured by the parties' indices of popularity (average percentage of electors represented on all the issues) and universality (percentage of cases when a majority is represented). These indices of representativeness are used instead of the conventional index 'number of votes received' to define the quotas of the parties in the Bundestag. This method is hypothetically applied to redistribute the Bundestag seats among the party factions in proportion to their indices, producing a considerable gain in the representativeness of the Bundestag. Finally, we discuss mixed election procedures combining the elements of conventional voting schemes with the Third Vote.

In Section 2 'Architectures of election procedures and their philosophy', two ways of collecting and processing electoral data and their philosophical backgrounds are compared.

In Section 3, 'Three concepts of representation — three votes', an alternative election method to enhance policy representation, the Third Vote, is hypothetically applied to redistribute the 2021 Bundestag seats, resulting in a significant gain in its representativeness.

In Section 4, 'Equalization effect and the Third Vote+', it is explained why the Third Vote tends to equalize the party quotas and why it is not harmful. To obtain a more habitual quota ratio (if really desired), a modification of the Third Vote — the Third Vote+ method — is described.

In Section 5, 'Combining the Third Vote with conventional election methods', it is described how the first and second votes can be used together with the Third Vote, thereby implementing in elections the concepts of descriptive, agent and policy representation.

[1] Figure 1 shows Pearson correlations for party votes in absolute figures, whereas [Tangian 2022a, Table 4] and [Tangian 2022b, Table 3] show Spearman rank correlations, moreover, for the votes of the parties except for *Gesundheitsforschung* and *Gartenpartei* that did not answer any *Wahl-O-Mat* question. Therefore, the correlation Votes–Representativeness here is not the same.

In Section 6, 'Coalitions in the 2021 Bundestag under the Third Vote', the effect of the Third Vote is examined with regard of the coalition building.

Section 7, 'Summary: Enhancing policy representation', the main findings are recapitulated and put into context.

2 Architectures of election procedures and their philosophy

Taking into account the adduced reasons, the goal of the Third Vote election method is to redirect the electorate's attention away from candidates as personalities and parties as ideological symbols toward their specific abilities to speak on behalf of the public and represent public opinion on various policy issues. In other words, the question '*Whom* are we electing?' is to be replaced by '*What* are we electing?' If a certain decision, like Brexit, is voted on, then the attention is redirected from the emotional toward rational aspects characterized by specific policy implications.

Before we go on, some remarks should be made on the *architecture* of election procedures — the word is used by analogy with 'software architecture' in computer science, i.e. the way and the order in which information is processed [Tangian 2003, p. 37]. Let us illustrate alternative architectures and their impact on the election outcome using two examples, going back to Ostrogorski's paradox [Nurmi 1999, pp. 70–73]; see also [Gehrlein and Lepelley 2011, pp. 123–124].

Example 1 (Two Architectures of Data Processing in Brexit Referendum) *Brexit — a portmanteau of 'British' and 'exit' for the withdrawal of the United Kingdom from the European Union — was put on a 2016 referendum, in which 51.9% voted to leave. According to the mass media, Brexit has three equally important effects: immigration restriction, closing the Irish border and economic recession/pound devaluation. These policy implications, at first overlooked by most people, were realized only as the British government failed to agree an acceptable Brexit scenario. The society was polarized and the government wallowed in endless debates and negotiations with the European Union. All of these led to a prolonged political crisis [Brexit 2019].*

Let us suppose that Table 1 describes the tolerance for the three Brexit effects in three equally large groups. The first group is negative regarding all the three consequences, but the second and third groups appreciate immigration restriction but differ in their attitudes toward other issues.

Table 1: Two architectures of voting for Brexit/No Brexit

Brexit implications	Tolerance profiles				Majority vote	Public profile
	1	2	3			
Immigration restriction	$-$	$+$	$+$	$\to$		$+$
Closing the Irish border	$-$	$+$	$-$	$\to$		$-$
Economic recession/Pound devaluation	$-$	$-$	$+$	$\to$		$-$
	$\downarrow$	$\downarrow$	$\downarrow$			$\downarrow$
Votes for Brexit	$-$	$+$	$+$	$\to$	$+$	
Public attitude toward Brexit						$-$

The individual determination is implemented in the conventional vote: each individual aggregates his/her opinion profile and makes his/her choice for or against Brexit. The corresponding election architecture has the order of operations $\downarrow\downarrow\downarrow \to$ *. Then two of the three groups vote for Brexit (*$+$*), and Brexit is approved (*$+$*).*

Under the public determination, the balances of public opinion on each issue are determined. The corresponding election architecture assumes the order of operations $\rightrightarrows \downarrow$ *. The resulting public profile consists of public attitudes toward each policy implication: one positive and two negative. After the aggregation of the public profile, Brexit is rejected (*$-$*).*

Thus, we can imagine that the Brexit crisis might have be avoided if the Brexit referendum were based on the policy representation concept instead of the conventional Yes/No-vote.

Now we consider a more complex example, illustrating how the election architecture can influence the choice among three candidates.

Example 2 (Two Architectures of Electoral Data Processing for Three Candidates) *Let three candidates for president, A, B and C, and three equal elector groups, 1, 2 and 3, position themselves on three questions: Reform the health care system?—Yes/No, Create more public jobs?—Yes/No, and Increase corporate taxes?—Yes/No as displayed in Table 2. The match-up of candidate positions with the electoral groups is displayed in Table 3. For instance, on Question 1, A represents the opinion of Elector groups 1 and 2, and the opposite opinion of Elector group 3 is represented by B and C. Hence, regarding Question 1, A is supported by 2/3 of the electorate. If the first architecture with the order of operations $\downarrow\downarrow\downarrow \rightarrow$ is applied, then each electoral group makes its own choice and casts votes for its favorite candidate. Then the Elector group 1 votes for candidate A, because A represents its opinion on three out of three issues, whereas C represents it on only one issue, and B on no issue at all. The other two electoral groups vote for B, who represents their opinion on two issues, whereas A and C represent them on one issue each. Under this architecture, B wins with 2/3 of the votes. This is the functionality implemented in VAAs (voting advice applications), e.g. Wahl-O-Mat, which finds the party with the closest political profile to that of the user [Bundeszentrale für politische Bildung 2021]. Here, each elector votes as if having been advised by a VAA.*

Table 2: Positions of elector groups 1, 2, 3 and candidates A, B, C on three issues

Question	Elector group's profiles			Candidate's profiles		
	1	2	3	A	B	C
1. Reform the health care system?	+	+	−	+	−	−
2. Create more public jobs?	+	−	+	+	−	−
3. Increase corporate taxes?	+	−	−	+	−	+

Table 3: Two architectures of election procedure for candidates A, B and C

Question	Matching candidates with elector groups				Majority	Public
	1	2	3		vote	match-up
1. Reform…	A	A	B,C	$\rightarrow$		A
2. Create…	A	B,C	A	$\rightarrow$		A
3. Increase…	A,C	B	B	$\rightarrow$		B
	$\downarrow$	$\downarrow$	$\downarrow$			$\downarrow$
Electors' votes	A	B	B	$\rightarrow$	B	
Public choice						A

If the second architecture with the order of operations $\rightrightarrows \downarrow$ is applied, then the representative of the prevailing public opinion is found for each issue. The first two rows of the table say that, on the first two issues, candidate A represents 2/3 of electors, whereas candidates B and C represent the remaining 1/3 of them. On the third issue, about corporate taxes, A and C represent 1/3 of electors, whereas B represents 2/3 of the electors. Then A is elected, because A represents the prevailing public opinion on two out of three issues, B represents it on one issue, and C on no issue. This architecture, differing from that of VAAs, is used in our indices of popularity and universality. Indeed, as follows from Table 2, the

electorate's majority opinions on the issues constitute the vector

$$\vec{a} = \begin{pmatrix} + \\ + \\ - \end{pmatrix},$$

and the frequencies with which candidates represent majority opinions (= the candidates' universality indices) are equal to $\mathsf{U}_A = 2/3,\ \mathsf{U}_B = 1/3,\ \mathsf{U}_C = 0$. *Hence, candidate A, with the highest universality, is the public choice.*

The two collective choice architectures illustrated by these examples are backed up by very different political philosophies. The first architecture denoted by $\downarrow\downarrow\downarrow\rightarrow$ reflects the liberal philosophy of *individual determination*. It is based on individualism in opinions and on understanding the public good as the sum of the good of every individual, in the spirit of John Locke:

> Every man has a 'property' in his own 'person.' This nobody has any right to but himself...
>
> The public good, i.e. the good of every particular member of that society.
>
> [Locke 1689, *Second Treatise of Government*, Ch 5, 26, and *First Treatise of Government*, Ch 9, 92]

This philosophy deals with the aggregation of what Rousseau and Condorcet called individual wills. Electors choose their favorite candidates themselves, according to their own criteria and without being asked why they cast votes for this or that candidate. They can be motivated by personal sympathies or by egoistic intentions rather than by the public good. Even if electors are given some reference criteria, as in our examples, they are free to ignore them or assign arbitrary weights. The privacy in expressing individual wills is incorporated in the electors' votes which are considered 'black boxes'.

The second architecture with the order of operations $\rightrightarrows\downarrow$ reflects the philosophy of *public determination*. It explicitly articulates the public interest, formulating socially important questions and asking for the electors' opinions on them. The society is considered a single body that has a political profile regarding these issues, resembling the Rousseauvian *general will*:

> Each of us puts his person and all his power in common under the supreme direction of the general will, and, in our corporate capacity, we receive each member as an indivisible part of the whole.
>
> At once, in place of the individual personality of each contracting party, this act of association creates a moral and collective body, composed of as many members as the assembly contains votes, and receiving from this act its unity, its common identity, its life and its will. This public person, so formed by the union of all other persons, formerly took the name of *city*, and now takes that of *Republic* or *body politic* ...
>
> There is often a great deal of difference between the will of all and the general will; the latter considers only the common interest, while the former takes private interest into account, and is no more than a sum of particular wills...
> [Rousseau 1762, *Of Social Contract*, Book I, 6 and Book II, 3]

The public political profile is used to find the most socially adequate candidate. This architecture enhances the civic aspect of election and reduces the partiality of electors' opinions. It is often used for evaluating new products, project proposals, scientific contributions, etc., when each referee estimates each quality separately. For instance, journal articles can be judged by the degree of innovation, awareness of recent literature, and presentation style, with the final decision being based on the collective evaluation of each quality rather than on referee votes.

The two architectures resemble two ways of aggregation in multiple elections [Brams et al. 1998]. Multiple election is selection from bundles of accepted/rejected propositions, e.g. (Create new jobs, Reduce student fees), (Create no new jobs, Reduce student fees), etc. Under the *combination aggregation*, which resembles the first architecture, each elector votes for one of the bundles, and the bundles are ranked by the votes received. In other words, each elector votes for a choice option — composite alternative, controversial candidate, etc. Under the *proposition aggregation*, which resembles the second architecture, each proposition is voted on separately, and the desired bundle is determined. If it is not available, the one that is closest to it is selected. In other words, electors vote for components of the alternatives/characteristrics of the candidates, then the combination of most desired components/characteristics is taken as a reference, and the closest among the available alternatives is selected.

Currently, the mathematical problem of how to best satisfy a collective with a composite program is studied within a new branch of social choice theory called *judgment aggregation* [Grossi and Pigozzi 2014, Lang et al. 2017, List 2012, List and Puppe 2009]. In a sense, our approach is in line with it, having relevance to some other fields as well. For instance, the voters' Yes/No answers to a question imply choosing several candidates (parties) at a time regarding the given issue — it is clearly seen from both [Tangian 2022a, Figure 1], and Example 2. Thereby, the operation on every question of the second architecture resembles *approval voting*, where electors are allowed to vote for all acceptable candidates [Brams and Fishburn 1982, Laslier and Sanver 2010].

Finally, it should be noted that the practice of applying voting wherever possible results in the predominant use of the first architecture and the frequent overlooking of the second one, even in cases where it could be more relevant. For instance, the Condorcet winner is found by means of voting procedures, which embody the philosophy of individual determination, whereas he is imagined as the best with regard to the general will. The individualistic approach is also seen in the definition of social utility as the sum of individual utilities, although it is not necessarily adequate. Probably, in both cases the second (public) architecture would be more relevant.

3 Three concepts of representation — three votes

The conventional vote by name uses the first election architecture, reflecting the philosophy of individual determination. The second architecture which operationalizes the philosophy of public determination can be implemented as well. The aims are: (a) redirecting the voters' attention from candidate (party) images to their manifestos and policy proposals, and (b) enhancing policy representation. The latter is attained by matching the candidate's policy profiles with that of the electorate. To be specific, we hypothetically modify the German Bundestag election procedure.

At first we focus on computing the parties' weights in the Bundestag, and we consider the problem of allocating the whole number of seats to factions at the end of the section.

3.1 Adding a third vote to German ballots

The German two-vote electoral system embodies two major historical concepts of political representation. The descriptive concept — the parliament portrays the society in miniature — is realized by the first vote *(Erststimme)*, with which local candidates are elected within constituencies and delegated to the federal parliament (Bundestag). These direct mandate holders from 299 German constituencies fill 299 Bundestag seats.

The agent concept — the parliament is an assembly of political experts who are the people's trustees and not necessarily their countrymen or fellows — is embodied in the second vote *(Zweitstimme)* for a party. The second vote has two functions: (1) qualifying parties to have the Bundestag seats — those

who receive nationwide at least 5% of the second votes,[2] and (2) determining the parties' shares of the Bundestag — in proportion to the second votes, including the direct mandate holders. For this purpose, another 299 Bundestag seats or more are allocated. Obviously, the second vote is decisive, because it determines the size of the party factions.

If the required proportion between party factions is unattainable within the regular $299+299=598$ seats, some extra seats are added. Finally, the Bundestag seats are allocated to eligible parties using the Sainte-Laguë/Scheper method; see [Tangian 2022c]. In the 2021 Bundestag elections, the greatest mismatch was between the CSU's 45 direct mandates and its too-small 5.66%-quota in the Bundestag derived from the second votes. To bring 45 direct mandates in line with the 5.66%-quota, the Bundestg had to be enlarged to 736 seats. For this purpose, 138 extra seats were given to other parties [Bundeswahlleiter 2021].

In the 1960s, academics started to discuss *policy representation* — how well the party system and the government represent policy preferences of the electorate. In 1989, this question had been unknowingly operationalized in the pioneering Dutch VAA (voting advice application) *StemWijzer* (= VoteMatch), which was at first distributed on diskettes and in 1998 launched on the internet [ProDemos 2021]; since then, the concept has been appropriated by several other countries [Garzia and Marschall 2014, Garzia and Marschall 2016]. VAAs find the party that best represents the user's policy preferences, thereby embodying the concept of policy representation at the level of single voters. Since VAAs are addressed to individuals, the implementation of this concept at the level of the whole of electorate remains beyond their scope.

The Third Vote fills in this gap. Let us imagine that the German ballots with their two votes are supplemented by the third vote *(Drittstimme)*, which is a questionnaire like in the *Wahl-O-Mat*: 'General speed limit on motorways?'—Yes/No, 'Germany should increase its defense spending?'—Yes/No, etc., and the parties have answered these questions during their electoral campaign. Then the ballots are processed and the parties are evaluated by an electoral procedure with either the first (individualistic) or the second (public) architecture as described in Section 2. We shall discuss these options in the next sections.

To summarize, the first vote, as before, is cast by name for the Bundestag member from the local constituency. The second vote — for a party — is used to reject unpopular or untrustworthy parties who receive fewer than 5% of the votes. Then the second vote retains only the filtering function, and its second function — determining the party weights in the Bundestag — is conveyed to the third vote. As we see, the partiality of the first vote for a person is reduced by defining quotas in the Bundestag according to the more impersonal second vote for a party. This logic of increasing impartiality of votes is continued in the absolutely impartial third vote.

The parties' policy profiles are backed up by certain ideologies, making the answers to different questions strongly interdependent. Therefore, just a few questions can provide a reliable basis for specifying the political profiles of both parties and electors. In a sense, we speak of a 'direct democracy test' — a competitive public examination of the parties, which are evaluated through a special election procedure with embedded referenda. As usual, the 'examination' can be based on a few key questions that suffice to make a general judgement.

Combining elections with referenda is practiced in Switzerland, Canada, the United States and some other countries, where it is done to avoid multiple campaigns for the convenience of the population [Referendum 2021]. Basing election on referenda is a step further: not only to let the electorate vote on particular actions of the authorities, but to form these authoritative bodies with respect to public opinion and general political context. When representatives are examined using referenda, the electorate gains more control over policy making.

[2] A party which has at least three direct mandates or represents an ethnical minority is also eligible for Bundestag seats even if it has less than 5% of the second votes.

Table 4: The 2021 indices of the Bundestag parties, the Bundestag and the hypothetical Bundestag as if elected using Third Vote methods

	P		U		Max quota ratio	1st vote (direct man-dates)		2nd vote (by party name)		3rd vote $\frac{P+U}{4}$		3rd vote+ $\frac{P+U}{4}-\frac{1}{2}$		1st vote & 3rd vote	1st vote & 3rd vote+	2nd vote & 3rd vote	2nd vote & 3rd vote+
	u	g	u	g		A	N	A	N	A	N	A	N	N	N	N	N
	%	%	%	%			%	%	%	%	%	%	%	%	%	%	%
SPD	59	60	71	72		121	40	26	28	65	17	15	33	29	37	22	30
CDU/CSU	56	58	62	67		143	48	24	26	61	15	11	23	32	35	21	24
GRÜNE	54	54	53	54		16	5	15	16	54	14	4	8	9	6	15	12
FDP	45	47	45	45		0	0	11	13	46	12	−4	0	6	0	12	6
AfD	50	52	58	62		16	5	10	11	56	14	6	12	10	9	13	12
DIE LINKE	55	55	56	55		3	1	5	5	55	14	5	11	8	6	10	8
SSW	52	53	62	61		0	0	0	0	57	14	7	15	7	7	7	7
Absolute Maximum	69	69	100	100													
BUNDESTAG by																	
2nd vote	57	59	63	65	5.6												
3rd vote	60	62	79	79	1.4												
3rd vote+	59	60	71	72	4.1												
1st vote/3rd vote	59	61	76	78	5.3												
1st vote/3rd vote+	58	60	74	75	6.2												
2nd vote/3rd vote	60	61	76	77	3.1												
2nd vote/3rd vote+	59	60	71	72	5.0												

3.2 Advised Vote for individual determination

The first (individualistic) election architecture, shown in Table 3 as $\downarrow\downarrow\downarrow\rightarrow$, assumes processing each individual ballot, whereupon the best matching party receives a vote. The election looks as operating the *Wahl-O-Mat* (or any other VAA), which finds the optimal party for every elector and casts the corresponding vote itself. This mode simulates the behavior of an individual who unconditionally trusts the VAA and follows its advice slavishly. Under this architecture, the election procedure remains traditional: votes are cast for parties and they are tallied as usual. Since this election method is intermediated by the VAAs' operation, it is called *Advised Vote*.

It should be noted that this election method, when applied to choosing the best candidate (party), inherits all paradoxes of voting. Indeed, it assumes the aggregation of numerous preferences and, in particular, can result in the election of the least preferable candidate (Borda paradox) or cyclic majorities (Condorcet paradox); see [Tangian 2020, Chapter 4]. Nevertheless, this method makes sense. Unlike just voting for a seemingly credible candidate, as it is done in conventional elections, the optimal choice of a multifaceted representative is a rather difficult task. Therefore, a rational assistance can be useful.

To illustrate rigorously this approach using the 2021 *Wahl-O-Mat* questions, we would need the electors' individual policy profiles. Since they are unavailable, we do it most simplistically, assuming that the voters have expressed their multi-criteria evaluations through the votes they actually cast. Hence, we obtain the distribution of the Bundestag seats as it is — elected by the second votes. Column '2nd vote‖A' in Table 4 displays the absolute percentages of the second votes received by the eligible parties, and Column '2nd vote‖N' shows these percentages normalized after the votes for non-eligible parties have been excluded, i.e. the quotas of the parties in the Bundestag. The Bundestag popularity and universality indices computed in [Tangian 2022a], for unweighted and Google-weighted questions are shown in Row 'BUNDESTAG by…2nd vote' and depicted at the beginning of the second row of Figure 2, where the bar length of each party is proportional to its quota in the Bundestag.

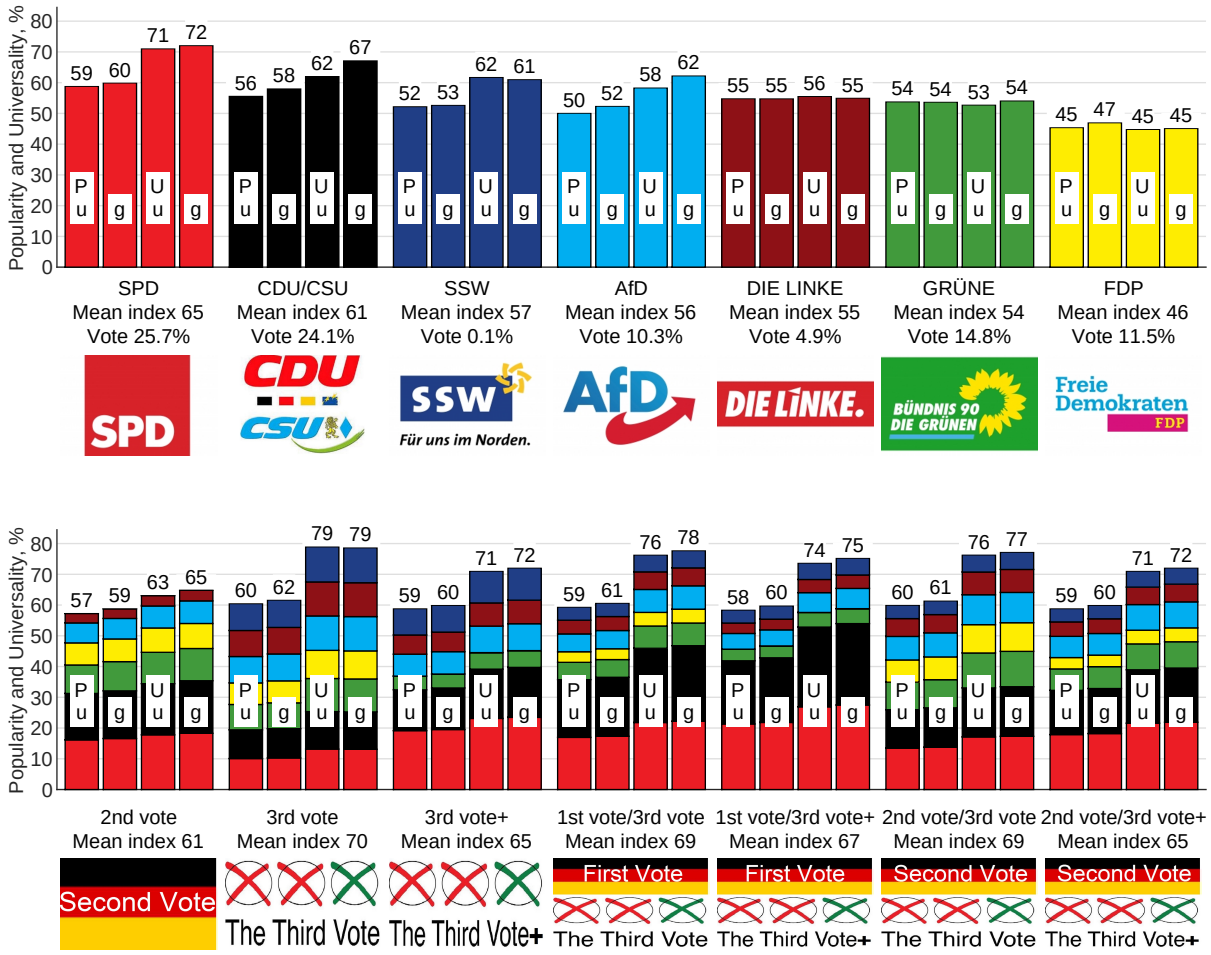

Figure 2: Visualization of Table 4. The 2021 indices of the Bundestag parties, the Bundestag and the hypothetical Bundestag as if elected using Third Vote methods: P—popularity, U—universality, u—for unweighted questions, and g—for Google-weighted questions.

3.3 Third Vote for public determination

The second architecture, shown in Table 3 as $\rightrightarrows\downarrow$, implements the philosophy of public determination. Under this architecture, the parties receive no votes even indirectly. Instead, the third votes — answers to a questionnaire — are used to construct the policy profile of the electorate with the balance of public opinion on each issue. Then the eligible parties are indexed — not with respect to the percentage of votes received but with respect to their policy representation capability — to reflect how well their policy profiles match with that of the electorate as a single body. For this purpose, we use our indices of popularity (the average percentage of the population represented) and/or universality (frequency in representing a majority).[3] Then the parties' Bundestag weights are defined in proportion to the parties' indices. Throughout the book, this election method is called the *Third Vote* and denoted '3rd vote'.[4]

Unlike the Advised Vote, the Third Vote does not lead to paradoxes. According to the philosophy of public determination, the Third Vote does not deal with numerous individual preferences but considers a single 'general will'. Obviously, a single preference ordering is not exposed to paradoxes.

[3]If desired, other measures of proximity between policy profiles can be defined.

[4]To avoid misunderstanding: the third votes (in small letters) are questionnaires in electoral ballots which are used either in the Advised Vote or the Third Vote election methods (capitalized).

We illustrate this approach using the 2021 Wahl-O-Mat questionnaire. As mentioned in the previous paragraph, the policy profiles of individual electors are unavailable, but they are not really necessary for our purpose. It suffices to know the profile of the electorate as a whole — which corresponds to the result of horizontal aggregation in Table 3. In [Tangian 2022a], the policy profile of the electorate is revealed through the relevant public opinion polls, and the degree of its matching with the parties' political profiles is measured by the mean of the parties' unweighted and Google-weighted popularity and universality indices. These indices are replicated in Table 4 and illustrated by Figure 2. The means of the four indices are given in Column '3rd vote $\frac{\mathsf{P}+\mathsf{U}}{4}\|$A'. The means normalized (reduced to the total of 100%) are shown in Column '3rd vote$\|$N', which suggests the hypothetical composition of the 2021 Bundestag as if elected by the Third Vote.

The popularity and universality indices of the Bundestag adjusted this way are displayed in Row 'BUNDESTAG by…3rd vote' and in the second row of Figure 2. As one can see, the representativeness of the 2021 Bundestag elected by the second votes is about 60%, meaning that it is 'more representative than non-representative', whereas the indices of the Bundestag as if elected by the Third Vote are much higher, being on the average $> 70\%$.

The Row *Absolute Maximum* in Table 4 indicates the theoretical maximum of the indices — when for all questions, only majorities in the society are represented; see [Tangian 2020, p. 284]. The absolute maximum provides a reference, showing to which extent the representativeness potential is exhausted. Since the index range is always symmetric about the central point of 50%, the index minimum follows from the maximum. For instance, the popularity maximum of 69% implies its minimum of 31%. Taking into account the range of 31–69%, the popularity's increase from 57–59% for the second vote to 63–65% for the Third Vote looks even more significant — not just 6 percent points but 16% of the range.

4 Equalization effect and the Third Vote+

4.1 Equalization effect

As one can see, the Third Vote equalizes the parties' quotas in the Bundestag. Indeed, the 2021 Bundestag's maximum quota ratio (neglecting the SSW) is equal to

$$\text{2nd vote maximum quota ratio SPD : LINKE} = 28 : 5 = 5.6\ ,$$

whereas for the Bundestag as if elected by the Third Vote it is equal to

$$\text{3rd vote maximum quota ratio SPD : FDP} = 17 : 12 \approx 1.4\ ;$$

see Table 4, Columns '2nd vote$\|$N' and '3rd vote$\|$N'.

The equalization effect is caused by the nature of the popularity and universality indices that determine the parties' quotas. Under conventional elections, political parties are voted on by disjoint groups of their adherents of very different sizes, implying the different sizes of the party factions, whereas under the Third Vote every party is also supported by the adherents of other parties who share the same position on the given issue. Instead of many groups of party adherents, on every issue there are only two quite numerous groups — Yes-group and No-group (not counting those who abstain), and the parties that share common views represent them jointly rather than exclusively. Correspondingly, the groups of party adherents join (differently on each issue), providing stronger support for weak parties. Such an extended account of the parties' policy representation capability, embodying the principle of proportional representation of public preferences in full, leads to a significant equalization of parties' quotas in the Bundestag. Therefore, the policy representation capability of big parties is not many times greated than that of small parties. Moreover, a significant presence of minor parties in the parliament contributes to

better representation of public opinion on numerous issues.[5]

By these reasons, the equalization effect should not be regarded harmful by itself. Moreover, it is accompanied by a gain in the parliament's policy representation. Therefore, from the viewpoint of policy representation, the equalization effect creates no problem at all.

In fact, there are two points, both beyond the policy representation we focus on here. The first is that the relations between parties and the electorate, understood in a broad sense, are not reduced to policy representation only. There are ideological, religious, ethnic, cultural and intellectual reasons which cannot be ignored. Large parties are much closer to the electorate due to networking, traditions, voters' self-identification with parties, candidates' political experience and trustfulness, etc., which should be reflected in the party quota ratio.

The second point is coalition building. Since the equalization effect can reduce the factions of major parties, a ruling coalition may require three or even more member parties, complicating negotiations or making them impossible at all. If a parliament majority attains no consensus, the work of the parliament can hardly be efficient.

Thus, there are reasons to moderate the equalization effect. As we show below, it can be done either by slightly modifying the Third Vote or by combining it with the conventional vote by party name.

4.2 The Third Vote+

To tackle the equalization effect by modifying the Third Vote, we reconsider the indices that determine the party quotas in the Bundestag. From the indices of popularity and universality, we remove their 'non-representative' segment and retain only the segment above the critical threshold of 50%. In other words, we focus on the positive representativeness only, and, correspondingly, define the party quotas in proportion to the parties' indices of *positive* representativeness. This modification of the Third Vote method is called *The Third Vote+* and denoted '3rd vote+'.

Computationally, the mean of the popularity and universality indices is reduced by 50%: Column '3rd vote‖A' in Table 4 is transformed into Column '3rd vote+‖A'. Then the negative values are nullified and the index is normalized (the total brought to 100%) — as shown in Column '3rd vote+‖N' — and the parties' quotas in the Bundestag are adjusted accordingly. Now the faction of FDP is nullified,[6] and the Bundestag's maximum party quota ratio is equal to

$$\text{3rd vote+}\quad \text{maximum quota ratio}\quad \text{GRÜNE} : \text{SPD} = 33 : 8 \approx 4.1\ ,$$

[5]The *Wahl-O-Mat*'s neglect of minor parties has been condemned by the Administrative Court of Cologne with the admonition that they must be given better chances:

> Just over a week before the European elections [on May 23–26,2019], the Federal Agency for Civic Education (bpb) has shut down its Wahl-O-Mat. According to a ruling of the Administrative Court of Cologne, the internet offer, which is intended to provide orientation in elections, may not be continued for the time being. In its current form, it violates the constitutionally guaranteed right to equal opportunities, the judges ruled, thereby giving an application by the party Volt Germany. This party felt disadvantaged because the Wahl-O-Mat users can only compare up to eight parties. Smaller and as yet unknown parties would be disadvantaged in this way, the court said, making it clear that the Federal Agency for Civic Education was obliged to give each party the same opportunities in the election campaign.
>
> [dpa/Zeit 2019a, Court stops the Wahl-O-Mat].

For more information see also [Ratzesberger 2019] and about the improved *Wahl-O-Mat* released a few days later see [dpa/Zeit 2019b].

[6]This purely theoretical count makes sense only if the elements of Column '3rd vote+‖A' in Table 4 are all positive, i.e. all Bundestag parties are more representative than non-representative. Otherwise, the nullification of factions for the parties that owe direct mandates must be overcome; possible solutions are suggested in the following Section 5.

which is even more contrasting than under the second vote. The popularity and universality indices of this version of Bundestag are displayed in Row '3rd vote+' of Table 4 and the second row of Figure 4. As one can see, the Bundestags determined by the 3rd vote and 3rd vote+, do not differ much in the popularity, but significantly differ in the quota ratio.

5 Combining the Third Vote with conventional election methods

5.1 Combining the Third Vote with direct mandates

In actuality, a party cannot have fewer seats than direct mandates. To make the Third Vote+ compatible with the first vote, we start with defining the parties' first-vote index. For this purpose, we normalize the number of direct mandates in Column '1st vote‖A', obtaining Column '1st vote‖N' in Table 4. Then we take the mean of Columns '1st vote‖N' and '3rd vote+‖N', obtaining Column '1st vote & 3rd vote+'. Such a mixed index is strictly positive for the parties with direct mandates and can be used to allocate the Bundestag seats retaining all the direct mandates. Indeed, (direct mandate's index + 3rd vote+ index)/2 guarantees retaining all 299 direct mandates in the twice larger Bundestag with 598 seats. We obtain a compatible combination of the first vote and the Third Vote+.

However, the resulting Bundestag allocation reserves no seat for the FDP, and the Bundestag's maximum quota ratio is equal to 6.2, greater than that for all the apportionments considered so far:

$$\text{1st vote/3rd vote+ maximum quota ratio SPD} : \text{GRÜNE} = 37 : 6 \approx 6.2 \ .$$

The popularity and universality indices of the Bundestag if elected this way are shown in Row 'BUNDESTAG by…1st vote/3rd vote+' of Table 4 and illustrated in the second row of Figure 2.

A more reasonable quota ratio is obtained if we combine the first vote and the Third Vote by applying the mean of the corresponding indices, which results in Column '1st vote & 3rd vote'. Now all the eligible parties receive Bundestag seats, and the maximum quota ratio is equal to

$$\text{1st vote/3rd vote maximum quota ratio CDU/CSU} : \text{FDP} = 32 : 6 \approx 5.3 \ .$$

The popularity and universality indices of the Bundestag adjusted accordingly are displayed in Table 4 and Figure 2.

It should be noted that, in spite of a high correlation between the number of party-identifiable first votes for persons and second votes for parties, the Bundestag is growing: the 2009, 2013, 2017 and 2021 Bundestags had 24, 33, 111 and 138 adjustment seats, respectively [Bundeswahlleiter 2009, 2013, 2017, 2021]. Since the answers to the Third Vote questionnaire bear no explicit party identification, the mismatch between the first vote and the Third Vote can be much stronger than between the first and second votes, requiring more significant Bundestag enlargements. This phenomenon is however of a quantitative rather than qualitative nature. The same can occur under the current election method, when a party has insufficient second votes to justify numerous direct mandates — so there is no contradiction with the existing rule. In [Tangian 2022c], a solution is suggested: relaxation of the principle 'one man—one vote' using adjustment vote weights for Bundestag members, which will always keep the Bundestag within the basic 598 seats.

5.2 Combining the Third Vote with vote by party name

The Third Vote can be combined with the conventional vote by party name. Again, this is attained by mixing the indices derived from the second and third votes. The mix can be done in any desired proportion, but we continue to use the simple mean. Column '2nd vote & 3rd vote' of Table 4 shows the mean of Columns '2nd vote‖N' and '3rd vote‖N'. Using this mixed index to define the party quotas in the Bundestag, we factually combine both voting methods in proportion 50 : 50.

Since every Bundestag party, being eligible, has normally a certain percentage of the second votes, the index in Column '2nd vote‖N' is strictly positive, making the mixed index also strictly positive. Then all the Bundestag factions have positive quotas as well. The popularity and universality indices of the resulting composition of the Bundestag are displayed in Row '2nd vote/3rd vote' of Table 4 and in the bottom row of Figure 2. As expected, the Bundestag's popularity and universality indices are intermediate between those in the table's rows 'BUNDESTAG by…2nd vote' and 'BUNDESTAG by…3rd vote'.

This combined election method inherits the equalization effect of the Third Vote. Indeed, as follows from Column '2nd vote & 3rd vote', the Bundestag's maximum quota ratio is equal to

$$\text{2nd vote/3rd vote} \quad \text{maximum quota ratio} \quad \text{SPD} : \text{SSW} = 22 : 7 \approx 3.1 \ .$$

To reduce the equalization effect, the conventional vote by party name can be combined with the Third Vote+. For this purpose, we take the mean of Columns '2nd vote‖N' and '3rd vote+‖N'. The result is shown in Column '2nd vote & 3rd vote+'. By the already adduced reasons, this index is strictly positive, excluding exceptional cases of nullification of Bundestag factions. The popularity and universality indices of this version of Bundestag are given in the bottom Row 'BUNDESTAG by…2nd vote/3rd vote+' of Table 4 and illustrated in Figure 2. The quota ratio approaches the maximum quota ratio under the second vote, being equal to

$$\text{2nd vote/3rd vote+} \quad \text{maximum quota ratio} \quad \text{SPD} : \text{FDP} = 30 : 6 \approx 5.0 \ .$$

6 Coalitions in the 2021 Bundestag under the Third Vote

Our discussion would be incomplete if limited to estimating only the Bundestag's representativeness before coalition building. Indeed, parliamentary factions usually unite into coalitions that finally determine the parliament decisions, so estimating the representativeness of possible coalitions is no less important than that of the Bundestag before the coalition building.

Following [Tangian 2022a, Section 3], we analyze minimum eligible coalitions in the 2021 Bundestags hypothetically elected by the seven methods considered in Table 4. For each composition of the Bundestag, Table 5 displays the characteristics of the most unanimous one — as the best compatible.

The table is visualized by Figure 3, which has the same design as [Tangian 2022a, Figure 3] except for that it displays the coalitions associated with alternative Bundestag compositions. As in the figure cited, the red flagstaff (additionally labeled '2') distinguishes the coalition of the Bundestag elected by the second vote, i.e. by party name. The green flagstaff indicates the Third Vote, and the blue one the Third Vote+. The mixed election methods are shown by dashed flagstaffs whose colors are associated with the election methods used. For instance, the black/green flagstaff indicates the combination of the first and third votes shown by black and green, respectively.

The representativeness of the most unanimous (i.e. most realistic) coalition depends on the way the Bundestag is elected. Ordering the non-mixed election methods (the first three in the table) by the coalition representativeness, we obtain the Third Vote+ (labeled '3+') at the top, then the Third Vote (labeled '3'), and then the second vote (labeled '2'). For the mixed methods, the order is as follows: the first vote and the Third Vote+ (labeled '13+') at the top, than the second vote and the Third Vote+ (labeled '23+') or the combination of the Third Vote with the first (labeled '13'), leaving behind the combination of the second and Third Vote (labeled '23').

The currently ruling coalition SPD+GRÜNE+FDP is missing in Table 5 and Figure 3 since its unanimity index is not the best under any election method considered (the unanimity depends on the election method, because the election method determines the party quotas, and their ratio has a probabilistic impact on the coalition unanimity). The ruling coalition only appears in Table 6 and Figure 4, where for each election method all minimum eligible coalitions with the unanimity $> 40\%$ are displayed. The

Table 5: Unweighted indices of the 2021 Bundestag most unanimous minimum eligible coalitions for seven election methods and faction weight factor $f = 0.5$

Election method Coalition	Faction/coalition weights	Unanimity	Popularity		Universality	
			Expectation	Standard deviation	Expectation	Standard deviation
	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank
2 (2nd vote)						
CDU/CSU+FDP+AfD	26+13+11=50/7	65.79/1	49.63/7	±2.27/7	52.47/7	±4.62/2
3 (3rd vote)						
SPD+GRÜNE+LINKE+SSW	17+14+14+14=59/3	55.26/3	55.35/6	±2.04/6	58.51/6	±5.32/7
3+ (3rd vote+)						
SPD+GRÜNE+LINKE	33+8+11=51/5	65.79/1	57.02/3	±1.80/3	69.20/3	±4.60/1
13 (1st vote/3rd vote)						
SPD+CDU/CSU	29+32=60/2	63.16/2	58.07/2	±1.56/1	65.86/4	±4.92/4
13+ (1st vote/3rd vote+)						
SPD+CDU/CSU	37+35=72/1	63.16/2	58.09/1	±1.56/2	71.03/1	±4.92/5
23 (2nd vote/3rd vote)						
SPD+GRÜNE+LINKE+SSW	22+15+10+7=54/4	55.26/3	55.79/5	±2.03/5	62.94/5	±5.30/6
23+ (2nd vote/3rd vote+)						
SPD+GRÜNE+LINKE	30+12+8=50/6	65.79/1	56.89/4	±1.81/4	69.61/2	±4.64/3

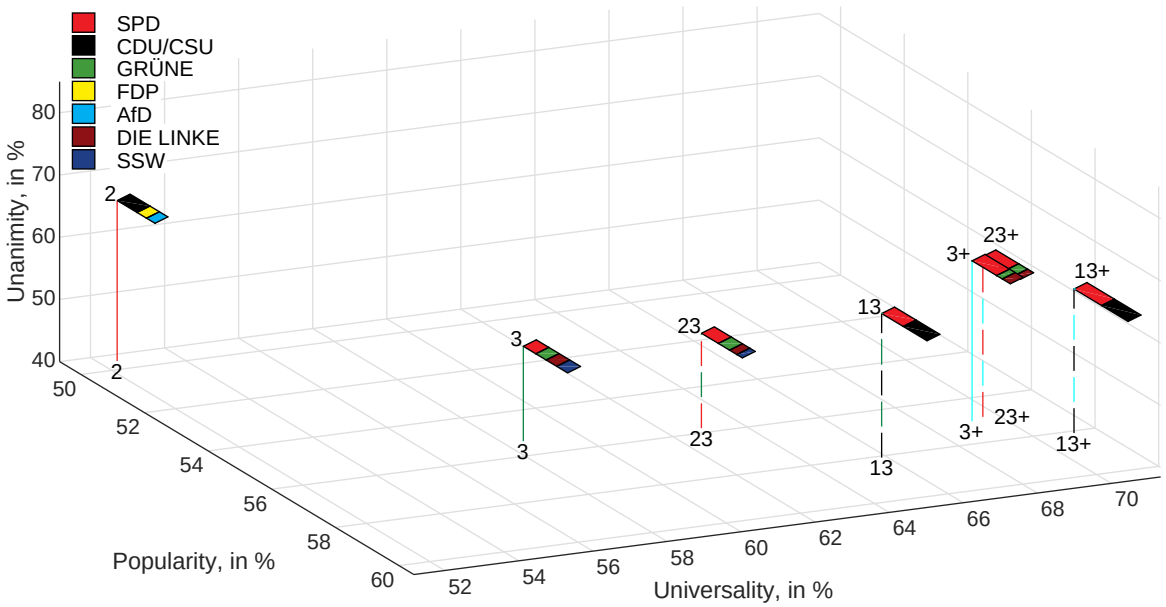

Figure 3: Visualization of Table 5. Election method by flagstaff: red — 2 (2nd vote), green — 3 (3rd vote), blue — 3+ (3rd vote+), black/green — 13 (mix of 1st and 3rd votes), black/blue — 13+ (mix of 1st vote and 3rd vote+), red/green — 23 (mix of 2nd and 3rd votes), red/blue — 23+ (mix of 2nd vote and 3rd vote+).

Table 6: Unweighted indices of the 2021 Bundestag minimum eligible coalitions with unanimity $> 40\%$ for seven election methods and faction weight factor $f = 0.5$

Election method Coalition	Faction/coalition weights	Unanimity	Popularity		Universality	
			Expectation	Standard deviation	Expectation	Standard deviation
	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank
2 (2nd vote)						
CDU/CSU+FDP+AfD	26+13+11=50/17	65.79/1	49.63/17	±2.27/15	52.47/17	±4.62/2
SPD+CDU/CSU	28+26=55/10	63.16/2	58.10/3	±1.56/4	71.01/3	±4.92/8
SPD+GRÜNE+FDP	28+16+13=57/5	44.74/5	53.21/16	±2.39/17	68.70/10	±5.87/15
3 (3rd vote)						
SPD+GRÜNE+LINKE+SSW	17+14+14+14=59/3	55.26/4	55.35/14	±2.04/13	58.51/15	±5.32/14
3+ (3rd vote+)						
SPD+GRÜNE+LINKE	33+8+11=51/13	65.79/1	57.02/6	±1.80/8	69.20/9	±4.60/1
SPD+CDU/CSU	33+23=55/7	63.16/2	58.16/1	±1.55/1	70.81/5	±4.90/5
SPD+GRÜNE+SSW	33+8+15=55/9	60.53/3	56.15/11	±1.79/7	68.43/11	±4.94/10
SPD+LINKE+SSW	33+11+15=58/4	60.53/3	56.29/10	±2.03/11	69.96/6	±4.97/11
13 (1st vote/3rd vote)						
SPD+CDU/CSU	29+32=60/2	63.16/2	58.07/5	±1.56/3	65.86/13	±4.92/7
SPD+GRÜNE+LINKE+SSW	29+9+8+7=53/12	55.26/4	56.14/12	±2.02/10	69.57/8	±5.25/12
13+ (1st vote/3rd vote+)						
SPD+CDU/CSU	37+35=72/1	63.16/2	58.09/4	±1.56/5	71.03/2	±4.92/9
SPD+GRÜNE+SSW	37+6+7=50/15	60.53/3	56.51/9	±1.74/6	67.93/12	±4.83/4
23 (2nd vote/3rd vote)						
SPD+GRÜNE+LINKE+SSW	22+15+10+7=54/11	55.26/4	55.79/13	±2.03/12	62.94/14	±5.30/13
SPD+CDU/CSU+FDP	22+21+12=55/6	44.74/5	55.22/15	±2.39/16	58.45/16	±5.98/17
SPD+CDU/CSU+SSW	22+21+7=51/14	44.74/5	56.54/8	±2.23/14	71.86/1	±5.91/16
23+ (2nd vote/3rd vote+)						
SPD+GRÜNE+LINKE	30+12+8=50/16	65.79/1	56.89/7	±1.81/9	69.61/7	±4.64/3
SPD+CDU/CSU	30+24=55/8	63.16/2	58.13/2	±1.56/2	70.91/4	±4.92/6

coalition SPD+GRÜNE+FDP appears only once — under the elections by the second vote being only the third in the list, i.e., being the least compatible.

It should be emphasized that the real-world coalition building does not exclusively rely on the closeness of the parties' positions, which in our model is measured by the unanimity index. There are also other important criteria, like ideological and personal compatibility, party image, size and influence, cooperation experiences, and some others. Therefore, it makes sense to consider a larger selection of minimum eligible coalitions, whose unanimity is not necessarily the best for each Bundestag composition, but still acceptable.

7 Summary: Enhancing policy representation

As we have discussed, the two historical concepts of political representation — the descriptive one and the agent one — are not well coordinated with the more recent concept of policy representation. To be implemented, the latter needs a dedicated election method that could make representative democracy 'more representative'. To meet this objective, we consider the Third Vote election method, which explicitly takes into account the voters' policy preferences. A hypothetical application of the Third Vote and its combinations with the first and second votes to re-allocate the 2021 Bundestag seats demonstrates a significant gain it its representativeness.

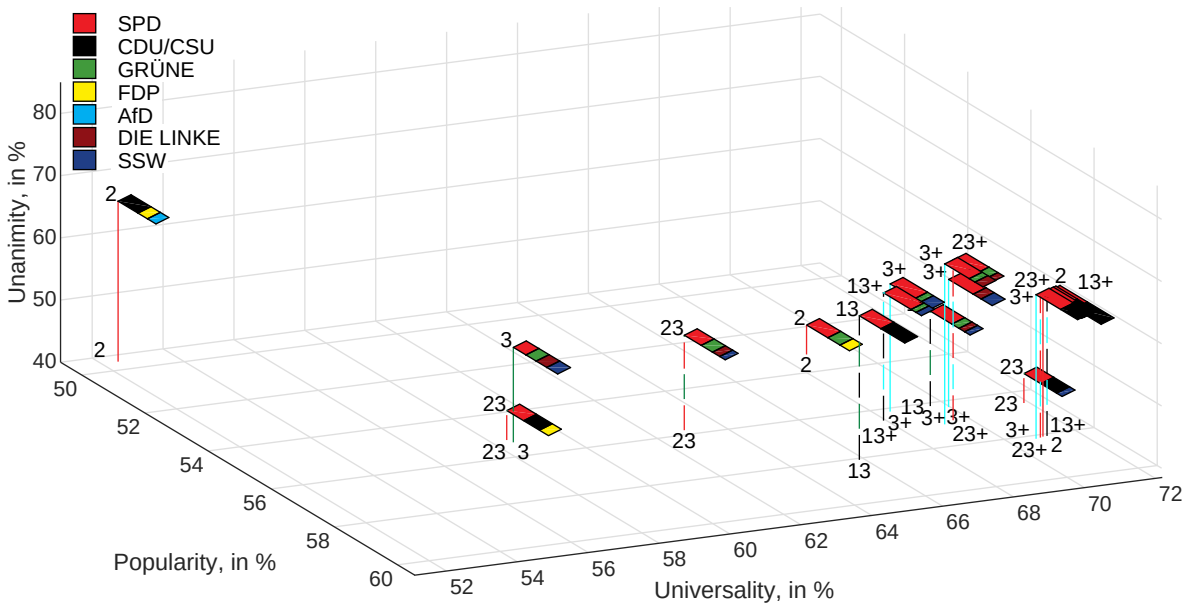

Figure 4: Visualization of Table 6. Election method by flagstaff: red — 2 (2nd vote), green — 3 (3rd vote), blue — 3+ (3rd vote+), black/green — 13 (mix of 1st and 3rd votes), black/blue — 13+ (mix of 1st vote and 3rd vote+), red/green — 23 (mix of 2nd and 3rd votes), red/blue — 23+ (mix of 2nd vote and 3rd vote+).

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