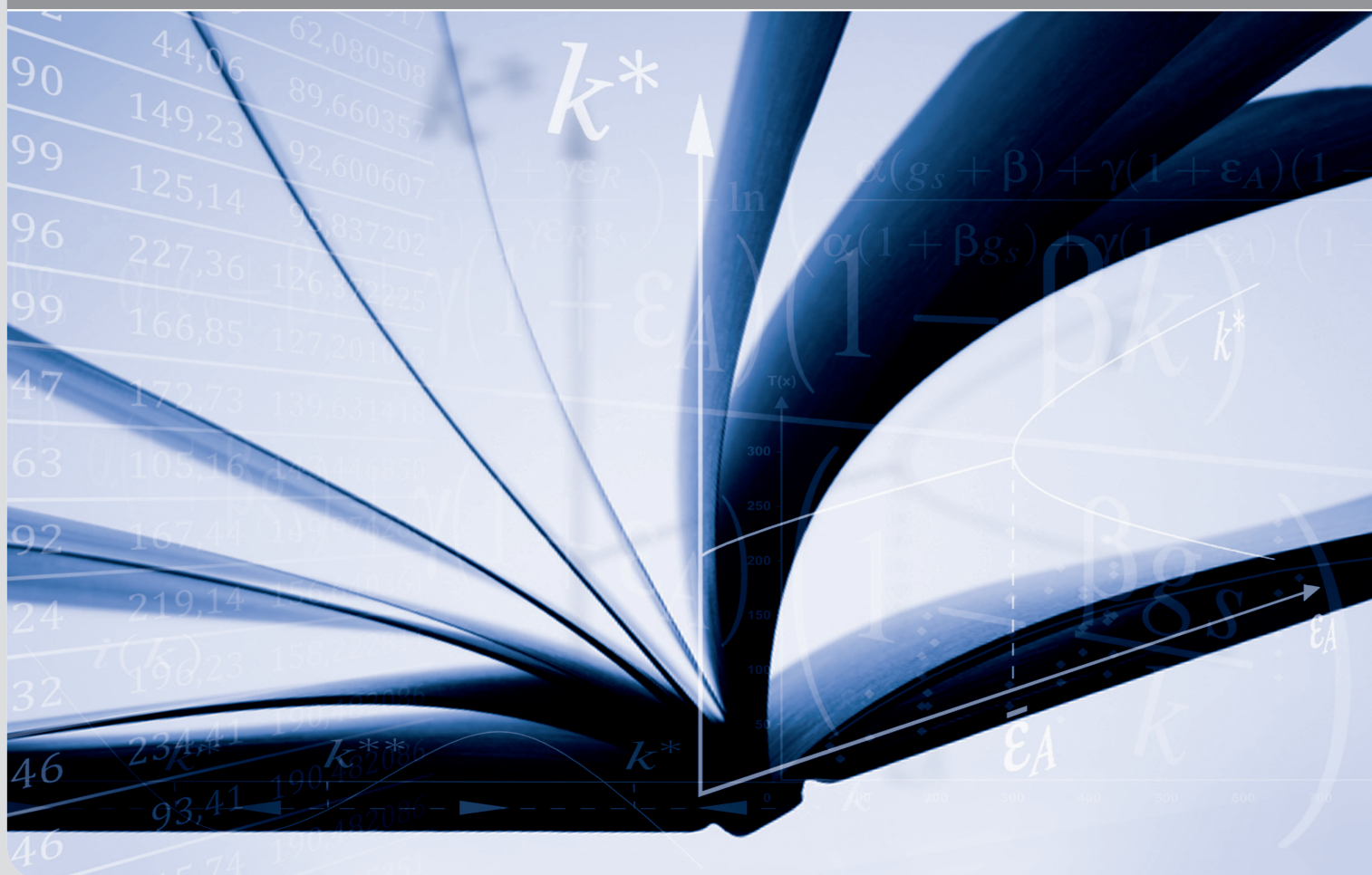


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Auctions versus Posted Prices with Costly Entry and Outside Options*

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Abstract

The decline of auction-format sales in favor of posted prices on digital marketplaces is commonly attributed to behavioral biases or secular changes in the market environment. This paper examines the latter by jointly modeling two forms of buyer opportunity cost within a standard symmetric independent private values framework: an entry cost sunk upon auction participation, and a mechanism-independent outside option reflecting the surplus a buyer can obtain from a close substitute at a known market price. A seller chooses between a second-price auction and a posted price; potential buyers decide whether to participate after observing their private valuations. The outside option endogenously partitions buyers into low-value types, who bid their full valuation, and high-value types, whose bids are capped at the outside option price, giving rise to qualitatively distinct entry regimes that the seller anticipates and actively shapes. Pressure from either channel strictly erodes the auction's advantage, but the seller's response to the two is asymmetric. The model yields testable implications that qualitatively align well with observed patterns and place weight on the outside option channel as a driver for the shift towards posted prices.

Keywords: auctions, posted prices, entry costs, outside options, mechanism design, e-commerce

JEL Codes: D44, D47, D82, L81

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1 Introduction

Over the past two decades, auction sales have declined markedly in favor of posted prices or hybrid mechanisms across many digital marketplaces. On eBay, once the paradigmatic online auction platform, the share of auction-format listings has fallen steadily since the mid-2000s, with posted prices now accounting for the vast majority of listings. This shift has attracted considerable attention in the literature. Existing explanations span both the seller and the buyer side. On the seller side, the most common argument points to the scalability of posted prices and the growing presence of large, commercial sellers on platforms that were originally dominated by individual sellers. This account has merit, but it is unlikely to tell the whole story: [Einav et al., 2018](#) document that the shift from auctions to posted prices also occurred within narrowly defined seller-product groups, ruling out compositional change as the sole driver. On the buyer-side, explanations have typically pointed to behavioral mechanisms: early auction platforms may have benefited from overbidding by inexperienced buyers, a phenomenon sometimes termed “auction fever” [Heyman et al., 2004](#), [Adam et al., 2011](#), while the introduction of hybrid formats such as buy-it-now prices has been interpreted as catering to risk-averse buyers who prefer price certainty [Hidvégi et al., 2006](#), [Reynolds and Wooders, 2009](#). A common thread in these accounts is the reliance on behavioral biases or departures from standard rational-agent assumptions to explain why the auction format has lost its appeal.

This paper offers an alternative explanatory framework on the buyer side that operates entirely within the bounds of rationality and risk-neutrality. It rests on two distinct buyer-side frictions that differ fundamentally in their scope. The first is an *entry cost*, capturing auction-exclusive transaction costs such as monitoring effort, time commitment, and the risk of losing. Its effect on the relative attractiveness of auctions is well established in the literature, and it is by construction a burden that only the auction format imposes: participating in an auction means committing effort to an uncertain and deferred allocation, whereas a posted price simplifies participation and purchase into a single immediate transaction at a known price, leaving no participation decision to be paid for separately. The entry cost therefore never enters the buyer’s problem under posted prices, and any increase in c tilts the comparison mechanically in favor of the posted price.

The central contribution of this paper is the introduction of a second channel that lacks precisely this asymmetry: an *outside option* that is available to buyers regardless of which selling mechanism is chosen. When the same or a similar good can be found and purchased elsewhere at a known price $\bar{p}$, any buyer who acquires the good from the seller forgoes the surplus they could have obtained from this alternative source. Crucially, a more attractive

outside option disfavors the auction even though it applies equally under any selling format - as the results show, its bite falls disproportionately on the auction because it dampens entry incentives and compresses bids.

This paper studies both channels jointly and shows that, despite their distinct economic origins, the outside option affects the seller’s auction revenue and strategic behavior in a fashion that is strikingly similar to the entry cost: both can be understood as opportunity costs of interacting with the seller’s sale mechanism, with the entry cost representing the opportunity cost of *participating* and the outside option representing the opportunity cost of *winning*.

Judging from available data, there seems to be considerably more evidence for a secular improvement in outside options than for a change in entry costs. The effective cost of finding and acquiring a substitute has been falling for decades: advances in information technology lower search costs [Bakos, 1997, Backus et al., 2014, Jolivet and Turon, 2019] and platforms invest heavily in optimizing search and recommendation algorithms [Dinerstein et al., 2018]. Furthermore, the substantial increase of product variety in E-Commerce (cf. [Hinz et al., 2011, Brynjolfsson et al., 2025]) strongly suggests that the differentiation costs when choosing among alternatives has been declining steadily with the maturing of online marketplaces.

On the entry cost side, the picture is more ambiguous. In absolute terms, auction interfaces have become more streamlined, mobile bidding has reduced monitoring effort, and platform-level innovations such as proxy bidding and automatic notifications have arguably lowered the raw cost of auction participation relative to earlier periods. However, the relevant benchmark is not the absolute level of auction friction but its magnitude relative to the increasingly frictionless alternative of buying at a posted price. Streamlining purchase processes and cutting delivery times has become a central value proposition of e-commerce platforms, and posted-price transactions have benefited from these improvements to a far greater extent than auction participation. As a result, even if auction entry costs have fallen in absolute terms, the relative hassle of participating in an auction might plausibly have increased.

The model developed in this paper formalizes these two channels within a standard symmetric independent private values (SIPV) framework. A seller chooses between a posted price and a second-price sealed-bid auction with a reservation bid. Potential buyers observe their private valuations and then decide whether to participate; under the auction, participation requires paying the sunk entry cost c . The outside option is available under either sales mechanism and value-dependent, with its value given as $w_i = \max\{v_i - \bar{p}, 0\}$.

The analysis yields several results. First, the outside option endogenously partitions buyers into two types: all buyers bid their true net value, but the outside option drives a wedge between valuation and net value only for high-value buyers, so that bids track the valuation below $\bar{p}$ and are capped at $\bar{p}$ above it. This partition gives rise to two qualitatively distinct entry regimes - deterministic threshold entry and stochastic mixed-strategy entry - with the seller optimally adjusting the reservation bid to avoid the latter. The seller's optimization under deterministic entry produces an *extended virtual value* that nests the classical virtual value of Myerson (1981) as a special case and transparently decomposes the seller's trade-off into an information rent and an entry rent. Second, the comparative statics show that both an increase in c and a decrease in $\bar{p}$ strictly reduce the auction's expected profit and can tip the seller's preference toward posted prices. When the outside option price falls below the optimal frictionless reservation bid b_0^{opt} , the auction loses its revenue advantage for any level of entry costs, including $c = 0$: the outside option alone suffices to eliminate the auction's edge. Third, the seller's optimal response to these two pressures is asymmetric: the reservation bid adjusts proactively to rising entry costs but responds to a falling outside option price only once the auction has already deteriorated past the point where it remains the preferred mechanism. Finally, the model identifies the existence of a unique, upward-sloping switching boundary $\hat{c}(\bar{p})$ in the parameter space, above which the posted price dominates and below which the auction is preferred. This boundary provides a unified account of the mechanism shift: as $\bar{p}$ falls and c rises, the auction-preferred region contracts from both directions.

A qualitative comparison with the descriptive findings of Einav et al. (2018) suggests that the model's predictions are broadly consistent with observed patterns on eBay. The cross-category coexistence of auctions and posted prices - with auctions persisting for used and idiosyncratic goods but disappearing for standardized products - aligns closely with variation in the outside option across product categories: formats diverge along the dimension of substitute availability rather than along plausible differences in participation costs. More broadly, the model's comparative statics and the predicted asymmetry in seller responses point toward a world in which a secular improvement in buyers' outside options is the primary driver of the shift toward posted prices, while changes in the entry cost play a subordinate role. These comparisons are developed in detail in Section 7.

The remainder of the paper is organized as follows. Section 2 reviews the related literature on auctions with entry costs and outside options. Section 3 sets up the model. Section 4 characterizes buyer and seller behavior under posted prices. Section 5 derives the equilibrium under the auction format, including optimal bidding, entry, and the seller's choice of reservation bid. Section 6 compares the two mechanisms and establishes the switching boundary.

Section 7 discusses the results and their implications.

2 Literature Review

Entry Costs in Auctions

A substantial body of theoretical work examines auctions in which participation is costly. The existing literature can be broadly organized into two strands.

The first strand considers settings in which bidders make their participation decision prior to learning their valuations - so-called *ex ante* entry. McAfee and McMillan [1987] and Levin and Smith [1994] provide the seminal treatments of this setting under symmetric, homogeneous entry costs: bidders are *ex ante* identical, entry is stochastic, and a free-entry equilibrium prevails in which participation rates adjust until expected bidder profits are driven to zero. The resulting equilibrium is jointly determined by the entry cost and the distribution of buyer values, without any selection of higher-value buyers into participation. Moreno and Wooders [2011] extend this line of work to heterogeneous entry costs, showing that equilibrium entry is then governed by a threshold rule: only bidders whose entry cost falls below a cutoff value participate, introducing selection on the cost dimension while values remain unknown at the time of the participation decision.

The second strand - and the one more directly related to the present paper - considers *interim* entry: bidders first observe their private valuation and only then decide whether to bear the entry cost and participate in the auction. This timing structure means that entry itself serves as an endogenous screening device, with only sufficiently high-value buyers finding it profitable to participate. The seminal contribution is Samuelson [1985], who establishes the fundamental equilibrium structure: a threshold value $\tilde{v}$ emerges such that all buyers with $v_i \geq \tilde{v}$ enter the auction and all buyers with $v_i < \tilde{v}$ abstain. Menezes and Monteiro [2000] refine and extend Samuelson's framework by embedding it in a mechanism design approach and deriving the revenue-maximizing selling mechanism when entry is endogenous and governed by an interim participation decision. The resulting optimal mechanism generally features a positive reservation price that trades off entry deterrence against revenue extraction from those who do enter. Subsequent contributions further explore the properties of equilibrium entry in this setting. Tan and Yilankaya [2006] characterize conditions under which the entry equilibrium in a second-price auction is unique, as well as when asymmetric equilibria can arise alongside the symmetric threshold equilibrium. Celik and Yilankaya [2009], building on related ideas and taking a mechanism design perspective, characterize the

optimal selling mechanism with simultaneous and costly interim participation. [Cao et al. \[2014\]](#) generalize the setup by allowing bidders to be privately informed about both their valuations and their individual participation costs, thereby combining interim entry with heterogeneous participation costs and establishing existence and uniqueness conditions in this two-dimensional type space. A different approach on heterogeneity is introduced by [Kaplan and Sela \[2022\]](#), who study second-price auctions in which entry costs are privately known to each bidder while valuation types are commonly known. Finally, [Lu \[2009\]](#) examine optimal auction design in the interim entry setting with a specific focus on entry subsidies as a tool for the mechanism designer, allowing the seller to trade off entry promotion against revenue extraction from entrants.

The model of the present paper belongs to the interim entry tradition of [Samuelson \[1985\]](#), although some characteristics of the resulting entry structure yield similarity to the first strand of literature discussed. The key contribution relative to the existing literature in this strand is the introduction of an outside option that is directly linked to each buyer's private valuation, generating a richer entry structure. Specifically, the outside option creates a natural partition of buyers into two types - low-value buyers for whom no viable outside option exists, and high-value buyers who can always acquire the good at a fixed price $\bar{p}$ elsewhere - which in turn gives rise to two qualitatively distinct entry regimes: deterministic threshold entry and stochastic mixed-strategy entry, depending on the parametric configuration of the model.

Outside Options in Auctions

The theoretical literature on outside options in auction settings is less unified than the entry cost literature, some are looked at here, some in the next chapter though several strands speak directly to the model developed here.

The most immediate connection is [Kirchkamp et al. \[2009\]](#), who provide a formal characterization of bidding equilibria in sealed-bid auctions with outside options. Their main theoretical result establishes a translation principle: if bidders hold private valuations $V \sim F_V$ and private outside options $W \sim F_W$, the optimal bidding strategy can be derived by constructing net valuations $X = V - W$ with the convoluted distribution F_X and applying the standard equilibrium bidding formula to F_X in place of F_V . For the second-price auction, this directly yields $b_i^* = v_i - w_i$ as the weakly dominant bidding strategy. The present paper applies this framework under the special case of a value-dependent outside option, $w_i = \max\{v_i - \bar{p}, 0\}$, which introduces heterogeneity in the effective outside option across the bidder population and thereby generates the partition into low- and high-value buyer

types that drives the entry structure of the model.

Outside options arise naturally when multiple sellers compete for the same pool of buyers, as each seller’s offering constitutes an outside option relative to participation in a competing auction. Although the present analysis does not model inter-seller competition directly, this competitive logic underlies the outside option structure adopted here. McAfee [1993] and Peters and Severinov [1997] provide foundational analyses of this setting, showing that competition among auctioneers generates an endogenous outside option for each bidder - the expected payoff from visiting a rival seller - which pins down equilibrium reserve prices and drives auction outcomes toward posted-price equivalents. Burguet and Sákovics [1999] qualify these results for the case of imperfect competition: when only two sellers compete, the outside option created by the rival auction is insufficient to fully discipline reserve prices, and sellers retain market power. Virág [2010] bridges the finite and large-market cases by showing that equilibrium reserve prices converge to sellers’ costs as the number of competing auctioneers grows, with comparative statics confirming that a larger number of sellers strengthens the bidders’ outside option and progressively erodes auction rents.

Auctions versus Posted Prices

A central question in mechanism design is which selling procedure maximizes the seller’s expected revenue. The seminal contribution of Myerson [1981] characterizes the optimal auction within the class of all incentive-compatible mechanisms, establishing the virtual value as the key sufficient statistic for revenue-maximizing allocation. While the present paper does not pursue optimal mechanism design in this general sense - it restricts attention to two specific selling formats commonly observed on e-commerce platforms - Myerson’s framework provides an analytical foundation for the seller’s optimization problem under the auction format. In particular, the optimal reservation bid derived in this paper is expressed in terms of the virtual value, and Corollary 4 is a direct extension of Myerson’s classical construct to the setting with costly entry.

Wang [1993] provides one of the earliest systematic comparisons of auctions and posted prices as competing selling mechanisms. Building on a model of temporally staggered buyer arrivals governed by a Poisson process, he derives conditions under which each format is preferred by the seller as a function of the shape of the value distribution and the cost structure of both mechanisms. Kultti [1999] studies auction and posted-price markets within an evolutionary dynamics framework and establishes an equivalence result that depends on the degree of competition and on the relative proportions of buyers and sellers. Ziegler and Lazear [2003] likewise employ a Poisson-based arrival model, with their main feature being an auction-

exclusive outside option: the time commitment required by auction participation exposes bidders to alternative purchase opportunities that arrive during the wait. A single arrival rate jointly captures the resulting waiting cost and the availability of substitutes, yielding a friction that bears partial resemblance to both the entry cost and the outside option of the present paper and that likewise dampens the seller’s preference for the auction format.

A naturally related strand is the literature on auctions with buy-it-now prices, which studies hybrid mechanisms that combine auction and posted-price elements within a single sealed-bid selling format. [Hidvégi et al. \[2006\]](#) and [Reynolds and Wooders \[2009\]](#) both show that, under buyer risk aversion, augmenting the auction with a fixed buy-price option raises seller revenue relative to a pure auction, because the posted-price channel attracts high-value buyers who would otherwise bid cautiously or abstain.

[Chowdhury et al. \[2019\]](#) compare auctions and posted prices in the context of online marketplaces in an experimental paper. Their model features two behavioral biases: impatience, which is conceptually similar to an entry cost but departs strongly from the standard auctions-with-entry framework, and dissuasion, an outcome bias arising from earlier failure to acquire the good in the auction.

On the empirical side, two structural contributions address the observed coexistence of auctions and posted prices, and both rely on seller-side outside options as a central explanatory device. [Hammond \[2013\]](#) shows that the joint use of the two mechanisms can be rationalized by seller heterogeneity: sellers with more attractive outside options prefer the immediacy of a posted price, whereas sellers with weaker alternatives are more willing to bear the delay and uncertainty of an auction. [Bauner \[2015\]](#) affirms the role of seller-side outside options and additionally emphasizes the demand side, arguing that buyer preferences co-determine the preference structure for different mechanisms. Consistent with the buy-it-now literature cited above, his findings suggest that posted-price and buy-it-now formats disproportionately cater to more risk-averse buyers.

From a conceptual standpoint, perhaps the most closely related paper to the present analysis is [Einav et al. \[2018\]](#), who empirically study the shift from auctions to posted prices on eBay using a rich dataset of matched listings. Their framework features a parsimonious model of mechanism comparison in which buyers face two sources of friction: an auction-exclusive “hassle cost” and a mechanism-independent reserve utility. Although the hassle cost is framed as an entry cost, it operates as a uniform deduction from the buyer’s auction value - which is structurally equivalent to a symmetric, flat outside option in the sense of [Kirchkamp et al. \[2009\]](#) rather than a sunk participation cost. Both channels induce

a pressure toward posted prices in their model, but neither triggers a within-mechanism seller response: the optimal reserve price and posted price are invariant to changes in the friction parameters. Their structural estimation attributes significantly more weight to the auction-exclusive hassle than to the common reserve utility.

This modeling approach is likely driven by tractability, as the model is not the primary focus of their paper. It does, however, open up an interesting observation: if the most natural reduced-form specification of auction friction is structurally ambiguous between an entry cost and an outside option, disentangling the two mechanisms requires a model that embeds both with full microeconomic detail. The present paper takes this as its starting point. It provides a characterization of both the entry cost and the outside option within a standard auction-theoretic framework, distinguishes their effects on entry, bidding behavior, and the seller’s optimal choice, and derives testable implications for each channel. The model affirms the comparative statics intuitions of the reduced-form approach while generating additional predictions - in particular regarding seller responses and entry patterns - that allow the two channels to be separated. Several of the qualitative findings documented by Einav et al. (2018) lend themselves to a comparison with these predictions, and - divergent to the structural estimation model of the authors - fit very well into a world where the outside option is a main explanatory channel in the decline of eBay auctions. The respective examination is taken up in Section 7.

3 The Model

The model takes the perspective of a seller in a marketplace who offers a single unit of a good. The seller can choose between two pricing mechanisms:

1. A posted price p , at which any willing buyer may purchase the good immediately.
2. A second-price sealed-bid auction with a reservation bid b_0 , where the good is allocated to the highest bidder at the price of the second-highest bid, provided this bid meets the reservation bid.

The seller’s private value for the good is normalized to zero. Under either mechanism, the offer is observed by $n \geq 2$ potential buyers with single-unit demand. Each buyer i is risk-neutral and holds a private valuation v_i for the good, drawn independently from a distribution with cumulative distribution function $F(v)$, density $f(v)$, and support $[\underline{v}, \bar{v}]$.¹ To guarantee unique maximizers throughout the model, f is assumed to be log-concave over

¹The characteristics of the demand distribution are common knowledge to all agents.

the whole value range. This is a rather mild condition satisfied by a broad class of commonly used distributions.² As a consequence, F is strictly regular in the sense of Myerson (1981) - the virtual value $\phi(v) = v - (1 - F(v))/f(v)$ is strictly increasing.

In addition to this standard symmetric independent private values (SIPV) framework, the model introduces two features that are the focal points of the analysis: an outside option available to buyers under both mechanisms, and an entry cost that buyers incur when participating in the auction. These two elements are described in turn in the following subsections.

Numerical illustration. Several of the results below are accompanied by closed-form illustrations using a simple parametrization: $n = 2$ buyers and a uniform value distribution, $V \sim U(0, 1)$. This parametrization is chosen for its tractability and is used solely for expository purposes; all qualitative findings hold for the general case.

3.1 Outside Option

No matter which pricing mechanism is chosen by the seller, each buyer has access to an outside option that can always be activated in case the good is not acquired through the seller. In terms of timing, this means that the outside option can be accessed by any buyer at any point (also *after* the sale of the good has taken place), with the only restriction being that it is mutually exclusive to buying the good directly from the seller. The value of this outside option to buyer i is $w_i = \max\{v_i - \bar{p}, 0\}$, where $\bar{p}$ is the fixed price at which the buyer can acquire the same good from an alternative source. This price is symmetric and constant across all buyers. Note that compared to other models with a fixed outside option, this formulation yields an *individual* outside option value that is linked to the buyer's private valuation.

With this specification, two kinds of buyers can be distinguished. Buyers with $v_i < \bar{p}$ have no viable outside option: they will either acquire the good from the seller or not at all. These buyers will be called *low-value buyers* in the following. Conversely, buyers with $v_i \geq \bar{p}$ will always obtain the good from one source or the other; they are never willing to pay more than $\bar{p}$ for the seller's good, but will seek to obtain it below that price if possible. These buyers will be called *high-value buyers*.

The parameter $\bar{p}$ admits a broader interpretation than the price for an identical good from a competing seller. It may incorporate a search cost component or a differentiation cost component, in the sense that the outside option need not be the exact same product but

²This includes for instance the uniform, normal, exponential or logistic distribution; see Bagnoli and Bergstrom (2005) for a comprehensive catalogue.

a close substitute. To formalize this point, one can think of $\bar{p}$ as a price measure for a similar good, normalized onto the value distribution of the original good. As an example, suppose the outside option is a close substitute priced at p_S , and that its value distribution G is a leftward shift of F , so that $G(v - \varepsilon) = F(v)$ - meaning that every buyer values the substitute by ε less than the original. Transforming the substitute's price by the same margin, $\bar{p} = p_S + \varepsilon$, projects the preference structure onto the value distribution of the original good and preserves the interpretation of $\bar{p}$ as the effective cost of the best alternative.

3.2 Entry Cost

If the auction format is chosen by the seller as the pricing mechanism (and only then), any potential bidder incurs an entry cost c in order to participate. This entry cost is constant and symmetric across all potential buyers. In contrast to, e.g., [Levin and Smith 1994](#), this model features *interim* entry: the participation decision and the entry cost are due only after each buyer has observed their private valuation. Furthermore, unlike the refundable participation cost in [Einav et al. 2018](#), the entry cost here is sunk once the decision to participate is made.

The entry cost can be rationalized as capturing the effort associated with auction participation - the attention, time investment, and monitoring that bidding requires. However, it need not only reflect the costs of participation; it is also conceivable that auction entry carries some intrinsic utility, for instance a level of excitement from bidding or the prospect of obtaining a bargain. This connects to the concept of “auction fever” as discussed in the literature [Heyman et al. 2004](#), [Adam et al. 2011](#), a collective term for emotion-induced bidding behavior. Conceptually, a negative c - reflecting net enjoyment from participation - is therefore consistent with the framework, although the subsequent analysis restricts attention to $c \geq 0$.

3.3 Solution Concept

The market described above constitutes a two-stage game of incomplete information. In the first stage, the seller publicly announces a pricing mechanism - either a posted price p or a second-price auction with reservation bid b_0 - together with the corresponding parameter value. This announcement is observed by all potential buyers. In the second stage, buyers respond: under the posted price, each buyer makes a simple binary participation decision; under the auction, buyers simultaneously choose whether to incur the entry cost and, conditional on entry, submit a sealed bid. Since buyers hold private valuations drawn from

the commonly known distribution F , the auction stage constitutes a Bayesian game and is analyzed in terms of its Bayesian Nash Equilibrium (BNE).

The overall solution concept used is a Perfect Bayesian Equilibrium (PBE). The seller anticipates, for any given mechanism and parameter choice, the BNE that will be played in Stage 2, and selects the Stage 1 announcement that maximizes expected profit given this anticipated behavior. The analysis proceeds in three steps:

1. Characterization of optimal buyer behavior in Stage 2: the participation decision under posted prices, and the entry decision together with the optimal bidding strategy under the auction.
- 2.1 Derivation of the seller's optimal mechanism parameter in Stage 1: the revenue-maximizing posted price p^* and the revenue-maximizing reservation bid b_0^* , respectively.
- 2.2 Comparison of the seller's expected profit across the two mechanisms to determine the preferred pricing mechanism.

4 Posted Price Incentives

Under a posted price p , buyer behavior is straightforward. Buyer i purchases the good from the seller if and only if doing so yields non-negative net utility and no strictly preferable outside option is available, i.e. $v_i \geq p$ and $p \leq \bar{p}$. The outside option price therefore functions as an upper bound on the seller's feasible posted prices: any $p > \bar{p}$ would be dominated for every buyer by the outside option, yielding zero revenue.

Provided this constraint is not binding, the seller faces a standard monopoly pricing problem against the distribution of the highest buyer valuation. The probability of a sale at price p equals the probability that at least one of the n buyers holds a valuation weakly exceeding p , which is $1 - F_{(n)}(p) = 1 - F(p)^n$. The seller's expected profit is therefore:

$$\pi_P = p \cdot (1 - F_{(n)}(p)) . \quad (4.1)$$

The first-order condition for an interior optimum³ is

$$p^{**} = \frac{1 - F_{(n)}(p^{**})}{f_{(n)}(p^{**})} = \frac{1}{\lambda_{(n)}(p^{**})} , \quad (4.2)$$

³In the following, the two-star** superscripts are reserved purely for interior optima of the sellers' strategy variables. General equilibrium notation is conventionally marked by one-star* superscripts.

where $\lambda_{(n)}$ denotes the hazard rate of $F_{(n)}$.⁴ This condition admits a unique solution because $\lambda_{(n)}$ is strictly increasing, as shown directly below in Lemma 1(ii).

Lemma 1 (Hazard rate properties under log-concavity). *Under the assumption of a log-concave distribution density f :*

- (i) F has a strictly increasing hazard rate.
- (ii) The highest order statistic $F_{(n)}$ has a strictly increasing hazard rate.
- (iii) The hazard rate of $F_{(n)}$ is strictly lower than that of F :

$$\lambda_{(n)}(v) < \lambda(v) \quad \forall v \in (\underline{v}, \bar{v}), \quad n \geq 2.$$

Proof. Part (i) follows from Bagnoli and Bergstrom [2005], Corollary 2 (p. 448): strict log-concavity of f implies strict log-concavity of the reliability function $1 - F$, which is equivalent to the hazard rate $\lambda(v) = f(v)/(1 - F(v))$ being strictly increasing.

For part (ii), define $\gamma(v) = 1/\lambda(v) = (1 - F(v))/f(v)$ and $\gamma_{(n)}(v) = 1/\lambda_{(n)}(v)$. Formulating $\gamma_{(n)}$ and factorising its numerator yields:

$$\gamma_{(n)}(v) = \frac{1 - F(v)^n}{n F(v)^{n-1} f(v)} = \gamma(v) \cdot A(v), \quad A(v) = \frac{1}{n} \sum_{j=0}^{n-1} F(v)^{-j}.$$

Since $F(v) \in (0, 1)$, one has $A(v) \geq 1$ and $A'(v) \leq 0$ for all v . Differentiating:

$$\gamma'_{(n)}(v) = \gamma'(v) A(v) + \gamma(v) A'(v).$$

By part (i), $\gamma'(v) = -\lambda'(v)/\lambda(v)^2 < 0$ and trivially, $\gamma(v) > 0$. Therefore, the first term of the derivative is strictly negative while its second term is weakly negative, so $\gamma'_{(n)}(v) < 0$ and $\lambda'_{(n)}(v) > 0$.

For part (iii), the decomposition $\gamma_{(n)}(v) = \gamma(v) \cdot A(v)$ established in part (ii) yields the result immediately. For $n \geq 2$ and $F(v) \in (0, 1)$, the factor $A(v) = \frac{1}{n} \sum_{j=0}^{n-1} F(v)^{-j}$ satisfies $A(v) > 1$: the $j = 0$ term contributes exactly 1, while each of the $n - 1$ remaining terms $F(v)^{-j}$ exceeds 1 strictly. Therefore $\gamma_{(n)}(v) > \gamma(v)$, and taking reciprocals gives $\lambda_{(n)}(v) < \lambda(v)$. $\square$

⁴This condition is equivalent to the zero point of the virtual value function of F 's n -th order statistic, $\phi_{(n)}(p) = p - (1/\lambda_{(n)}(p))$.

5 Auction Incentives

Under the auction format, both the entry cost and the outside option shape buyer behavior. The outside option determines each buyer's net valuation of the auctioned good - and therefore the optimal bid - while the entry cost introduces a non-trivial participation decision. Together, these features generate an entry equilibrium that the seller must anticipate when choosing the reservation bid b_0 . This section characterizes optimal bidding and entry behavior; the seller's optimization problem is the subject of the subsequent subsection.

5.1 Optimal Bidding and Entry

5.1.1 Bidding Strategy

The following result is standard in the analysis of second-price auctions with outside options (cf. e.g. Kirchkamp et al. [2009]).

Lemma 2 (Weakly dominant bidding strategy). *In a second-price sealed-bid auction with reservation bid b_0 and outside option $w_i = \max\{v_i - \bar{p}, 0\}$, the weakly dominant bid for a participating buyer with valuation v_i is:*

$$b_i^*(v_i) = v_i - w_i = \min\{v_i, \bar{p}\}.$$

Proof. The standard dominant-strategy argument for second-price auctions applies with an adjusted valuation. A buyer who wins the auction forgoes the outside option, so the net value of winning is $v_i - w_i$. By the usual logic, each buyer optimally bids this net value. Since $w_i = 0$ for $v_i < \bar{p}$ and $w_i = v_i - \bar{p}$ for $v_i \geq \bar{p}$, the bid simplifies to $b_i^* = v_i$ for low-value buyers and $b_i^* = \bar{p}$ for high-value buyers. $\square$

The resulting bid function $b_i^*(v_i)$ is depicted in Figure 1: it follows the 45-degree line for low-value buyers ($v_i < \bar{p}$) and is flat at $\bar{p}$ for high-value buyers ($v_i \geq \bar{p}$).

For notational convenience, non-participating buyers are henceforth treated as having submitted a negative bid. Since $b_0 \geq 0$, such bids never win the auction, making this convention equivalent to non-entry.

5.1.2 Entry Decision

A risk-neutral buyer enters the auction if and only if the expected payoff from participation, net of the entry cost, weakly exceeds the outside option value. For a buyer with valuation

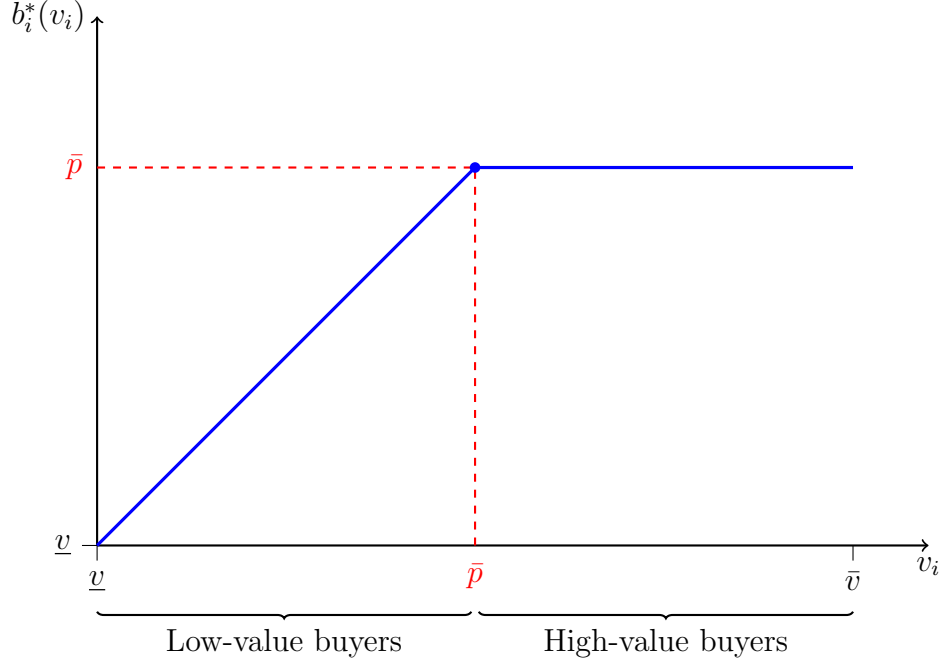


Figure 1: Optimal bid function in the second-price auction with outside options

v_i , optimal bid b_i^* , and outside option value w_i , entry is preferred whenever

$$\underbrace{P_{\text{win}}(b_i^*) \cdot (v_i - E[b_{(n-1)} \mid b_i^* = b_{(n)}]) + (1 - P_{\text{win}}(b_i^*)) \cdot w_i - c}_{\text{expected gross payoff from entry}} \geq w_i, \quad (5.1)$$

where $P_{\text{win}}(b_i^*)$ denotes the probability of winning with the optimal bid and $E[b_{(n-1)} \mid b_i^* = b_{(n)}]$ the expected payment conditional on winning (i.e. the maximum of the second-highest bid and the reservation bid b_0). Rearranging and substituting $v_i - w_i = b_i^*$ yields the reduced entry condition:

$$P_{\text{win}}(b_i^*) \cdot (b_i^* - E[b_{(n-1)} \mid b_i^* = b_{(n)}]) \geq c. \quad (5.2)$$

The left-hand side can be interpreted as the expected surplus from winning the auction relative to the outside option. Since the bid function has a kink at $\bar{p}$, the entry analysis divides naturally into two regimes. To build intuition, two polar cases are examined before stating the general result.

5.1.3 Special Case 1: All Buyers Are Low-Value ($\bar{v} \leq \bar{p}$)

If the entire support of valuations lies below $\bar{p}$, the outside option is worthless for all buyers: $w_i = 0$ and $b_i^* = v_i$ for every i . The model reduces to a standard second-price auction with entry costs, functionally equivalent to the procurement auction of [Samuelson \[1985\]](#).

In this setting, entry is governed by a threshold strategy. A critical buyer with valuation $\tilde{v}$ is indifferent between entering and abstaining; all buyers with $v_i > \tilde{v}$ strictly prefer entry, while those with $v_i < \tilde{v}$ stay out. For the critical buyer, winning requires that all $n - 1$ other buyers draw valuations below $\tilde{v}$ and therefore do not enter, so $P_{\text{win}} = F(\tilde{v})^{n-1}$. Conditional on winning as the sole entrant, the buyer pays the reservation bid b_0 , yielding a net gain of $\tilde{v} - b_0$. The indifference condition (5.2) therefore becomes:

$$F(\tilde{v})^{n-1} \cdot (\tilde{v} - b_0) = c . \quad (5.3)$$

5.1.4 Special Case 2: All Buyers Are High-Value ($\underline{v} \geq \bar{p}$)

If all valuations exceed $\bar{p}$, every participating buyer bids $b_i^* = \bar{p}$ by Lemma 2. Since all bids are identical, a crucial property emerges: whenever two or more buyers enter, the good is sold at price $\bar{p}$, and every buyer - winner and losers alike - attains a payoff of $v_i - \bar{p} = w_i$. Entry can therefore be profitable only if there is a positive probability of being the sole entrant and acquiring the good at $b_0 < \bar{p}$.

This structure has two further implications. First, the entry decision is independent of a buyer's private valuation: the payoff differential between entering and not entering depends only on the common parameters $\bar{p}$, b_0 , and c . Second, it rules out a symmetric pure-strategy entry equilibrium, since universal entry drives the surplus from participation to zero, which cannot cover the entry cost $c > 0$. The set of symmetric equilibria is therefore restricted to mixed strategies⁵

Let $\tilde{\delta} \in (0, 1)$ denote the common entry probability. A buyer who enters is the sole entrant with probability $(1 - \tilde{\delta})^{n-1}$, in which case they win the auction at price b_0 and obtain a gross payoff of $v_i - b_0$. With the complementary probability $1 - (1 - \tilde{\delta})^{n-1}$, at least one other buyer also enters; all entrants bid $\bar{p}$, and the entering buyer obtains w_i regardless of the auction outcome. The indifference condition between entering and taking the outside option is therefore:

$$(1 - \tilde{\delta})^{n-1} \cdot (v_i - b_0) + (1 - (1 - \tilde{\delta})^{n-1}) \cdot w_i - c = w_i . \quad (5.4)$$

Substituting $v_i - w_i = \bar{p}$, simplifying and solving for $\tilde{\delta}$:

$$\tilde{\delta} = 1 - \left(\frac{c}{\bar{p} - b_0} \right)^{\frac{1}{n-1}} . \quad (5.5)$$

⁵An asymmetric pure-strategy equilibrium in which exactly one predetermined buyer enters also exists. However, the symmetric mixed-strategy equilibrium is adopted as the focal solution, consistent with the symmetric and anonymous nature of the interaction.

5.1.5 General Case

The two special cases provide the building blocks for entry behavior when $\bar{p}$ lies in the interior of the value support, $\bar{p} \in (\underline{v}, \bar{v})$. The parameter determining the form of the equilibrium is $F(\bar{p})^{n-1} \cdot (\bar{p} - b_0)$: this is the expected surplus that a buyer at the boundary valuation $\bar{p}$ would earn from entering the auction if all buyers with valuations below $\bar{p}$ abstain. Comparing this benchmark to the entry cost c determines which regime prevails.

Proposition 1 (Entry equilibrium). *Under the maintained assumptions, the symmetric Bayesian Nash Equilibrium of the entry game takes one of two forms.⁶*

Case 1 (Deterministic threshold entry): *If $F(\bar{p})^{n-1} \cdot (\bar{p} - b_0) \geq c$, there exists a unique threshold $\tilde{v} \in (b_0, \bar{p}]$, implicitly defined by*

$$F(\tilde{v})^{n-1} \cdot (\tilde{v} - b_0) = c, \quad (5.6)$$

such that the entry strategy is

$$\delta_i^*(v_i) = \begin{cases} 1 & \text{if } v_i \geq \tilde{v}, \\ 0 & \text{if } v_i < \tilde{v}. \end{cases} \quad (5.7)$$

Case 2 (Stochastic entry): *If $F(\bar{p})^{n-1} \cdot (\bar{p} - b_0) < c$, low-value buyers do not enter and each high-value buyer enters independently with probability*

$$\tilde{\delta}_H = \frac{1 - \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{1}{n-1}}}{1 - F(\bar{p})}, \quad (5.8)$$

giving the entry strategy

$$\delta_i^*(v_i) = \begin{cases} \tilde{\delta}_H & \text{if } v_i \geq \bar{p}, \\ 0 & \text{if } v_i < \bar{p}. \end{cases} \quad (5.9)$$

Proof. Case 1. When $F(\bar{p})^{n-1}(\bar{p} - b_0) \geq c$, the entry surplus at the boundary valuation $\bar{p}$ weakly exceeds the entry cost. The expected surplus from entry, as given by the left-hand side of (5.2), is increasing in v_i for low-value buyers and constant across high-value buyers (since all high-value buyers bid $\bar{p}$). If the inequality is strict, a unique threshold $\tilde{v} < \bar{p}$ exists at which the critical buyer is just indifferent; all buyers above $\tilde{v}$ - including all high-value buyers - strictly prefer entry. The right-censoring of bids at $\bar{p}$ does not affect this calculation,

⁶Interior solutions are assumed throughout; boundary cases are omitted.

since any entrant with valuation above $\tilde{v}$ outbids the critical buyer regardless of whether their bid is censored. If the inequality holds with equality, no low-value buyer can cover the entry cost and the threshold is $\tilde{v} = \bar{p}$: all high-value buyers enter with certainty while all low-value buyers abstain. In both subcases, the critical buyer's participation constraint reduces to (5.3): winning requires being the sole entrant (probability $F(\tilde{v})^{n-1}$), and the reward is $\tilde{v} - b_0$.

Case 2. When $F(\bar{p})^{n-1}(\bar{p} - b_0) < c$, no low-value buyer can cover the entry cost. Among high-value buyers, the logic of Special Case 2 applies: all entrants bid $\bar{p}$, so entry decisions are independent of private valuations and the equilibrium is in mixed strategies. Each of the n buyers is independently of high-value type with probability $1 - F(\bar{p})$ and, conditional on being high-value, enters with probability $\tilde{\delta}_H$. The unconditional probability that any given other buyer enters the auction is therefore $(1 - F(\bar{p}))\tilde{\delta}_H$, and the probability of being the sole entrant is $(1 - (1 - F(\bar{p}))\tilde{\delta}_H)^{n-1}$. The indifference condition (5.4), applied with this sole-entrant probability, yields (5.8) after solving for $\tilde{\delta}_H$. $\square$

Several properties of this entry equilibrium are worthy of a remark. There is a continuous transition between the two regimes: at the boundary $F(\bar{p})^{n-1}(\bar{p} - b_0) = c$, which by convention falls under Case 1 with $\tilde{v} = \bar{p}$, the entry behavior coincides with the limiting case of Case 2 as $\tilde{\delta}_H \rightarrow 1$: all high-value buyers enter with certainty, all low-value buyers abstain. In both regimes, the expected number of entrants is decreasing in the entry cost c and in the reservation bid b_0 , by standard arguments. The regime itself is also steered by these parameters: increases in b_0 or c push toward stochastic entry, while an increase in $\bar{p}$ pushes toward deterministic threshold entry. Note that stochastic entry requires both $c > 0$ and $\bar{p} < \bar{v}$; in absence of either condition, the mixed-strategy regime cannot arise.

Figure 2 illustrates the transition across entry regimes as the entry cost varies. At $c = 0$, every buyer enters regardless of valuation. As c increases, entry remains deterministic but the threshold $\tilde{v}$ rises, excluding buyers with progressively higher valuations. Once $\tilde{v}$ reaches $\bar{p}$, the regime transitions to stochastic entry, with the entry probability $\tilde{\delta}_H$ declining further as c continues to increase.

5.2 Seller Incentives

The seller maximizes expected profit by choosing the reservation bid b_0 . This decision involves a fundamental trade-off: lowering b_0 encourages entry - thereby increasing the probability of a sale and of competitive bidding - but also reduces the revenue in the event that only a single buyer enters. Moreover, the level of b_0 determines which entry regime prevails,

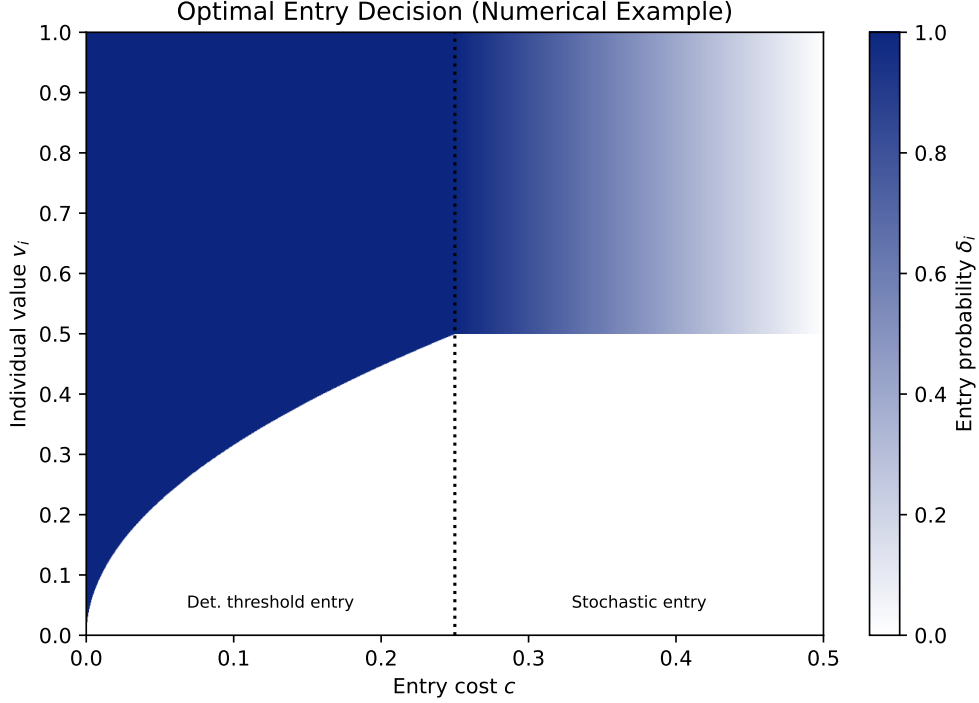


Figure 2: Optimal entry decision as a function of individual valuation and entry cost. Plotted with the parametrization of the numerical example, as well as $b_0 = 0$ and $\bar{p} = 0.5$

as established in Proposition 1. Since the entry equilibrium is continuous at the transition between the two regimes, the seller's profit function inherits this continuity. The analysis proceeds by deriving the optimal reservation bid under each regime in turn.

5.2.1 Deterministic Threshold Entry

Suppose the auction parameters admit deterministic threshold entry (Case 1 of Proposition 1). The seller's expected revenue decomposes into two events: either exactly one buyer enters and pays the reservation bid, or two or more buyers enter and the good is sold at the second-highest bid, which is censored from above at $\bar{p}$ by Lemma 2:

$$\pi_{A, \text{det.}} = P(v_{(n)} \geq \tilde{v} \wedge v_{(n-1)} < \tilde{v}) \cdot b_0 + P(v_{(n-1)} \geq \tilde{v}) \cdot E[b_{(n-1)} \mid v_{(n-1)} \geq \tilde{v}] . \quad (5.10)$$

Expanding the probabilities and accounting for the censoring of bids at $\bar{p}$:

$$\pi_{A, \text{det.}} = b_0 \cdot n F(\tilde{v})^{n-1} (1 - F(\tilde{v})) + \int_{\tilde{v}}^{\bar{p}} v \cdot f_{(n-1)}(v) dv + \int_{\bar{p}}^{\tilde{v}} \bar{p} \cdot f_{(n-1)}(v) dv . \quad (5.11)$$

The first term captures the revenue from a sole entrant; the second, the revenue when the second-highest valuation lies in the low-value range $[\tilde{v}, \bar{p})$ and the second-highest bid equals that valuation; the third, the revenue when at least two high-value buyers enter, in which case the price is capped at $\bar{p}$.

To characterize the optimum, it is convenient to re-parameterize the problem in terms of the entry threshold $\tilde{v}$ rather than b_0 directly. From the entry condition (5.3), b_0 and $\tilde{v}$ are linked by the bijective identity $b_0 = \tilde{v} - c/F(\tilde{v})^{n-1}$, so maximizing over $\tilde{v}$ is equivalent to maximizing over b_0 . Substituting this identity and differentiating with respect to $\tilde{v}$ yields (see Appendix A for the full derivation):

$$\frac{\partial \pi_{A, \text{det.}}}{\partial \tilde{v}} = n [F(\tilde{v})^{n-1} (1 - F(\tilde{v}) - \tilde{v} f(\tilde{v})) + c f(\tilde{v})] \quad (5.12)$$

Proposition 2 (Optimal reservation bid under deterministic threshold entry). *Under the maintained assumptions about F , the seller's profit under deterministic threshold entry admits a unique interior maximum. The first-order condition is*

$$\frac{c}{F(\tilde{v})^{n-1}} = \tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})} = \phi(\tilde{v}) \quad (5.13)$$

and the corresponding optimal reservation bid is

$$b_0^{**} = \frac{1 - F(\tilde{v})}{f(\tilde{v})} = \frac{1}{\lambda(\tilde{v})} \quad (5.14)$$

where $\lambda(\tilde{v}) = f(\tilde{v})/(1 - F(\tilde{v}))$ denotes the hazard rate of F .

Proof. Setting (5.12) to zero, dividing by $f(\tilde{v})$, and rearranging produces (5.13). Uniqueness follows because the left-hand side $c/F(\tilde{v})^{n-1}$ is strictly decreasing in $\tilde{v}$, while the right-hand side $\phi(\tilde{v}) = \tilde{v} - (1 - F(\tilde{v}))/f(\tilde{v})$ is strictly increasing according to Lemma 1. The optimal reservation bid is obtained by substituting the FOC into the entry condition $b_0 = \tilde{v} - c/F(\tilde{v})^{n-1}$. $\square$

The following lemma establishes that the optimal reservation bid under deterministic threshold entry moves inversely with the entry cost.

Lemma 3 (Monotonicity of the optimal reservation bid in the entry cost). *Under deterministic threshold entry, the optimal reservation bid b_0^{**} is strictly decreasing in the entry cost c .*

Proof. Rearranging the first-order condition (5.13) gives $c = \phi(\tilde{v}) \cdot F(\tilde{v})^{n-1}$. At the optimum, both factors are strictly positive and strictly increasing in $\tilde{v}$, so $\partial c / \partial \tilde{v} > 0$ and therefore $\partial \tilde{v}^* / \partial c > 0$. Since $b_0^{**} = 1/\lambda(\tilde{v}^*)$ by (5.14) and the hazard rate λ is strictly increasing (Lemma 1(i)), the chain rule yields:

$$\frac{db_0^{**}}{dc} = \frac{\partial b_0^{**}}{\partial \tilde{v}} \cdot \frac{\partial \tilde{v}}{\partial c} < 0 .$$

□

The structure of the first-order condition (5.13) can be interpreted through the lens of Myerson's virtual value. In the standard second-price auction without entry costs, the seller directly controls the entry threshold via $\tilde{v} = b_0$ and maximizes revenue by setting $\phi(\tilde{v}) = \tilde{v} - (1 - F(\tilde{v}))/f(\tilde{v}) = 0$, where the inverse hazard rate $(1 - F(\tilde{v}))/f(\tilde{v})$ represents the information rent that the seller concedes to ensure truthful bidding. In the present model, the seller still controls $\tilde{v}$ through b_0 , but now via the implicit entry condition (5.3). In addition to the information rent, the seller must account for the entry cost: the term $c/F(\tilde{v})^{n-1}$ on the left-hand side of the FOC can be interpreted as an *entry rent* - the compensation that the marginal entrant requires to cover the participation cost c . This leads to an extended virtual value function:

Corollary 4 (Extended virtual value). *The optimal entry threshold $\tilde{v}^*$ under deterministic entry satisfies $\phi_{\text{entry}}(\tilde{v}^*) = 0$, where*

$$\phi_{\text{entry}}(\tilde{v}) = \underbrace{\tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})}}_{\text{virtual value}} - \underbrace{\frac{c}{F(\tilde{v})^{n-1}}}_{\text{entry rent}} . \quad (5.15)$$

For $c = 0$, this reduces to the standard virtual value of Myerson [1981].

5.2.2 Stochastic Entry

Now suppose the auction parameters admit stochastic entry (Case 2 of Proposition 1). In this regime, high-value buyers enter with probability $\tilde{\delta}_H$ and are indifferent between entering and taking the outside option. This indifference property provides a convenient route to the profit function: since each entrant earns zero expected surplus relative to the outside option, the seller captures the entire transaction value $\bar{p}$ whenever the good is sold, net of the aggregate entry costs mustered by the buyers:

$$\pi_{\text{A, stoch.}} = \bar{p} \cdot P(k \geq 1) - n \cdot c \cdot \tilde{\delta}_H \cdot (1 - F(\bar{p})) , \quad (5.16)$$

where k denotes the number of buyers who enter and $P(k \geq 1) = 1 - (F(\bar{p}) + (1 - \tilde{\delta}_H)(1 - F(\bar{p})))^n$. Substituting the equilibrium entry probability (5.8) and simplifying yields:

$$\pi_{A, \text{stoch.}} = \bar{p} \left(1 - \left(\frac{c}{\bar{p} - b_0} \right)^{\frac{n}{n-1}} \right) - n c \left(1 - \left(\frac{c}{\bar{p} - b_0} \right)^{\frac{1}{n-1}} \right). \quad (5.17)$$

Proposition 3 (Optimal reservation bid under stochastic entry). *Under stochastic entry, the seller's expected profit is strictly decreasing in b_0 for all $b_0 > 0$. The optimal reservation bid is therefore $b_0^* = 0$.*

Proof. Differentiating (5.17) with respect to b_0 and rearranging:

$$\frac{\partial \pi_{A, \text{stoch.}}}{\partial b_0} = \frac{n}{n-1} \cdot \left(\frac{c}{\bar{p} - b_0} \right)^{\frac{n}{n-1}} \cdot \frac{-b_0}{\bar{p} - b_0}. \quad (5.18)$$

For $b_0 > 0$, every factor except $-b_0$ is strictly positive, so $\partial \pi / \partial b_0 < 0$. At $b_0 = 0$, the derivative vanishes. The seller therefore optimally sets $b_0^* = 0$. The detailed derivation is given in Appendix B. $\square$

The reservation bid affects the seller's profit through two competing channels. On the one hand, a higher b_0 raises the revenue in the sole-entrant event ($k = 1$), where the good is sold at the reservation bid. On the other hand, a higher b_0 tightens the entry condition and reduces the equilibrium entry probability, lowering the likelihood of any sale at all. Proposition 3 demonstrates that the second effect strictly dominates the first for every $b_0 > 0$: the marginal loss of entry probability always outweighs the marginal gain in sole-entrant revenue. The seller therefore pushes b_0 down at least until the equilibrium reaches the transitional point between both entry regimes, or to zero if that transitional point is not in reach.

The preceding two propositions jointly characterize the seller's optimal reservation bid in an intermediate parameter region where the unconstrained interior optimum is infeasible but deterministic entry can still be sustained.

Corollary 5 (Optimal reservation bid under constrained threshold entry). *Suppose $\phi(\bar{p}) \cdot F(\bar{p})^{n-1} < c < F(\bar{p})^{n-1} \cdot \bar{p}$. Then the seller optimally sets*

$$b_0^* = \bar{p} - \frac{c}{F(\bar{p})^{n-1}},$$

placing the entry equilibrium at the boundary of deterministic threshold entry with $\tilde{v} = \bar{p}$.

Proof. The lower bound on c implies that the unconstrained optimal threshold from Proposition 2 satisfies $\tilde{v}^* > \bar{p}$, so the interior optimum is infeasible within the deterministic regime. Since the deterministic profit is unimodal with its peak beyond the feasible range, it is increasing over the admissible set $b_0 \leq \bar{p} - c/F(\bar{p})^{n-1}$, and the constrained maximum is attained at the upper boundary. The upper bound on c ensures this boundary reservation bid is non-negative and denotes the limit at which stochastic entry cannot be prevented any more. By Proposition 3, stochastic profit is strictly decreasing in b_0 for $b_0 > 0$, so profit cannot be improved by raising b_0 past the transition into the stochastic regime. By continuity of the profit function across regimes, the transitional reservation bid is globally optimal. $\square$

The contents of the last section show that while the entry equilibrium from the buyers' perspective takes only two forms - deterministic threshold or stochastic (Proposition 1) - the seller's optimal reservation bid distinguishes three regimes: unconstrained threshold entry, where the interior FOC solution is feasible (Proposition 2); constrained threshold entry, where the seller lowers b_0 to the regime boundary (Corollary 5); and stochastic entry, where $b_0^* = 0$ (Proposition 3). The subsequent analysis treats these three cases separately.

Figure 3 illustrates the resulting optimal reservation bid as a function of c and $\bar{p}$, using the parametrization of the numerical example. The parameter space divides into four regions:

1. *Unconstrained threshold entry*: the seller sets b_0^* according to the FOC (5.13), with the entry threshold $\tilde{v}^*$ in the low-value range ($\tilde{v}^* < \bar{p}$).
2. *Constrained threshold entry*: the unconstrained optimum would require $\tilde{v}^* > \bar{p}$, so the seller adjusts b_0 downward until $\tilde{v} = \bar{p}$, the transitional point between both entry regimes.
3. *Stochastic entry*: no $b_0 \geq 0$ can sustain deterministic entry, so the seller sets $b_0^* = 0$ per Proposition 3.
4. *No viable auction*: below the line $\bar{p} = c$, the entry cost exceeds the maximum gain from entering even as the sole bidder with $b_0 = 0$, and no buyer participates.

6 Comparison of Pricing Mechanisms

With the seller's optimal behavior under both pricing mechanisms characterized, this section compares the two regimes and determines the seller's preferred mechanism as a function of the entry cost c and the outside option price $\bar{p}$. The analysis proceeds in three parts. First, the analysis of two baseline cases provides an initial assessment of seller preferences for two

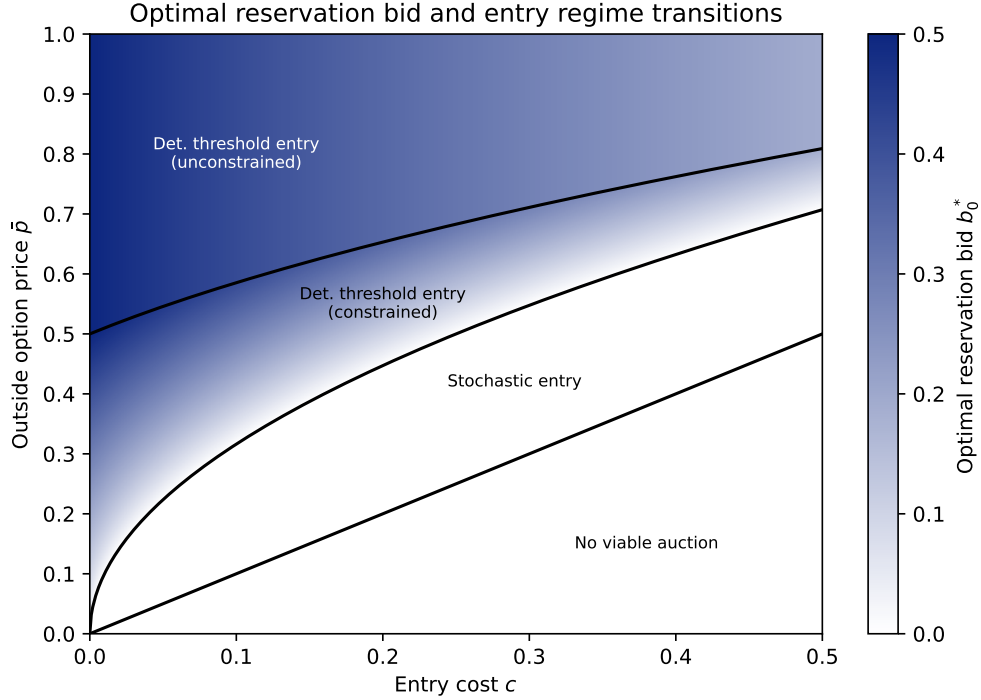


Figure 3: Optimal reservation bid and transition between entry regimes in the $(c, \bar{p})$ -space

polar scenarios. The second and third parts then examine the comparative statics effects of an increase in c and a decrease in $\bar{p}$, respectively, showing that either variation can erode the auction's attractiveness and ultimately tip the seller's preference toward posted prices. The last subsection summarizes the results.

6.1 Baseline Cases

For the first baseline case, assume that there are no entry costs ($c = 0$) and that the outside option is not viable ($\bar{p} \geq \bar{v}$). Under these conditions, the auction reduces to a standard second-price auction in which every buyer with $v_i \geq b_0$ participates and bids true to their full valuation. The following proposition establishes that the auction strictly dominates the posted price in this setting.⁷

Proposition 4 (Auction dominance in the standard case). *Let $c = 0$ and $\bar{p} \geq \bar{v}$. Then the seller's expected profit under the optimally designed auction strictly exceeds the expected profit under the optimal posted price:*

$$\pi_A(b_0^*) > \pi_P(p^*) .$$

⁷This result is a special case of the more general principle that the optimal auction maximizes expected revenue among all incentive-compatible mechanisms [Myerson, 1981].

Proof. Let $p^{**} \in (\underline{v}, \bar{v})$ denote the revenue-maximizing posted price. Consider an auction with reservation bid $b_0 = p^{**}$. Under this auction, a sale occurs if and only if at least one buyer has $v_i \geq p^*$ - exactly the same event that triggers a sale under the posted price. When exactly one buyer's valuation exceeds p^{**} , that buyer wins and pays $b_0 = p^{**}$, yielding the same revenue as the posted price. When two or more buyers have valuations above p^{**} , the auction generates additional revenue through competitive bidding, as the transaction price equals the second-highest bid rather than p^{**} . Formally:

$$\pi_A(b_0 = p^{**}) = \pi_P(p^{**}) + \int_{p^{**}}^{\bar{v}} (v - p^{**}) f_{(n-1)}(v) dv . \quad (6.1)$$

The integral is strictly positive because $n \geq 2$ ensures that $P(v_{(n-1)} \geq p^{**}) > 0$, i.e. there is positive probability that at least two buyers have valuations above p^{**} . Since b_0^{**} maximizes auction revenue by definition, the chain of inequalities follows:

$$\pi_A(b_0^{**}) \geq \pi_A(b_0 = p^{**}) > \pi_P(p^{**}) . \quad (6.2)$$

□

For the second baseline case, assume that $c > \phi(\bar{p}) \cdot F(\bar{p})^{n-1}$, so that by Corollary 5 the seller is constrained in the sense that his optimal reservation bid b_0^* induces entry at the deterministic boundary ($\tilde{v} = \bar{p}$) or stochastic entry (cf. Proposition 1), i.e. $F(\bar{p})^{n-1}(\bar{p} - b_0^*) \leq c$. Note that in the case of constrained threshold entry this relation holds with equality. The following proposition establishes that under these conditions, the posted price strictly dominates the auction.

Proposition 5 (Posted price dominance in the constrained case). *Let $c > \phi(\bar{p}) \cdot F(\bar{p})^{n-1}$, and therefore suppose the seller's optimal reservation bid b_0^* induces constrained threshold or stochastic entry. Then the seller's expected profit under the optimal posted price strictly exceeds the expected profit under the optimally designed auction:*

$$\pi_P(p^*) > \pi_A(b_0^*) .$$

Proof. The achievable posted price is potentially constrained by $\bar{p}$, so set $p^* = \bar{p}$ as a lower bound for the optimal posted-price profit:

$$\pi_P^* \geq \bar{p} (1 - F(\bar{p})^n) .$$

Under constrained threshold entry or stochastic entry, the auction profit at the optimal

reservation bid b_0^* is given by (5.17):

$$\pi_A^* = \bar{p} \left(1 - \left(\frac{c}{\bar{p} - b_0^*} \right)^{\frac{n}{n-1}} \right) - n c \left(1 - \left(\frac{c}{\bar{p} - b_0^*} \right)^{\frac{1}{n-1}} \right).$$

The entry regime condition requires $c/(\bar{p} - b_0^*) \geq F(\bar{p})^{n-1}$ which can be monotonously transformed to $(c/(\bar{p} - b_0^*))^{n/(n-1)} \geq F(\bar{p})^n$. Substituting into the first term yields

$$\pi_A^* \leq \bar{p} (1 - F(\bar{p})^n) - n c \left(1 - \left(\frac{c}{\bar{p} - b_0^*} \right)^{\frac{1}{n-1}} \right).$$

For the auction to attract any entrants, the entry cost must satisfy $c < \bar{p} - b_0^*$, which implies $(c/(\bar{p} - b_0^*))^{1/(n-1)} < 1$ and therefore makes the second term strictly negative. It follows that

$$\pi_A^* < \bar{p} (1 - F(\bar{p})^n) \leq \pi_P^*.$$

□

6.2 Comparative Statics - Entry Cost

Since the entry cost applies only to the auction, the posted-price profit $\pi_P(p^*)$ is invariant in c . The following proposition shows that auction profit, by contrast, is strictly decreasing in the entry cost under all entry regimes.

Proposition 6 (Monotonicity of auction profit in the entry cost). *For any $\bar{p} \in (c, \bar{v}]$, the seller's optimized auction profit $\pi_A^*(c)$ is strictly decreasing in c on the domain where the auction remains viable. In particular, $\pi_A^*(c) \rightarrow 0$ as $c \rightarrow \min\{\bar{p}, \bar{v}\}$.*

Proof. The argument proceeds in two parts.

Deterministic threshold entry. Suppose first that the parameters admit unconstrained threshold entry (Proposition 2). By Lemma 3, an increase in c raises the optimal entry threshold ($\partial \tilde{v}^*/\partial c > 0$) and lowers the optimal reservation bid ($\partial b_0^*/\partial c < 0$). Both effects strictly reduce auction profit: a higher threshold shrinks the set of entrants, lowering the probability of a sale and of competitive bidding, while a lower reservation bid reduces the revenue in the sole-entrant event. Under constrained threshold entry (Corollary 5), the entry threshold is fixed at $\tilde{v} = \bar{p}$ and the set of entrants is unchanged, but the reservation bid $b_0 = \bar{p} - c/F(\bar{p})^{n-1}$ is strictly decreasing in c , which directly lowers the sole-entrant

revenue. In both cases:

$$\frac{d\pi_{A, \text{det.}}^*}{dc} < 0 .$$

Stochastic entry. Now let the parameters of the auction be set in such a way that stochastic entry necessarily arises. Substituting the optimal reservation bid $b_0^* = 0$ (Proposition 3) into the profit function (5.17) and differentiating with respect to c yields:

$$\frac{\partial \pi_{A, \text{stoch.}}^*}{\partial c} = -n \left(1 - \left(\frac{c}{\bar{p}} \right)^{\frac{1}{n-1}} \right) . \quad (6.3)$$

This expression is strictly negative whenever $c < \bar{p}$, a condition that must hold for the auction to attract any entrants.

Since π_A^* is strictly decreasing in c under all regimes and continuous at their transition, the result holds globally. As $c \rightarrow \min\{\bar{p}, \bar{v}\}$, even the highest-valued buyer cannot cover the entry cost at $b_0 = 0$, so $\pi_A^*(c) \rightarrow 0$. $\square$

6.3 Comparative Statics – Outside Option

Under the posted price, the outside option (weakly) constrains the seller only when $\bar{p} \leq p^{**}$, in which case the outside option binds and $\pi_P^* = \bar{p}(1 - F(\bar{p})^n)$, so:

$$\frac{\partial \pi_P^*}{\partial \bar{p}} = 1 - F(\bar{p})^n - n \bar{p} f(\bar{p}) F(\bar{p})^{n-1} . \quad (6.4)$$

For $\bar{p} > p^{**}$, posted-price revenue is entirely unaffected by $\bar{p}$.

The auction is more immediately sensitive. By Lemma 2, every buyer with $v_i \geq \bar{p}$ bids exactly $\bar{p}$, so even when b_0^* remains at its interior optimum, a reduction in $\bar{p}$ clips the ceiling on competitive bids and reduces the revenue extracted from high-value buyers. This effect operates for any $\bar{p}$ in the interior of the value support, irrespective of whether $\bar{p}$ directly constrains the reservation bid.

Proposition 7 (Monotonicity of auction profit in the outside option price). *For any $c \geq 0$, the seller's optimized auction profit π_A^* is strictly increasing in $\bar{p}$ on the domain where the auction is viable. Specifically:*

(i) *Under unconstrained threshold entry (Proposition 2),*

$$\frac{d\pi_{A, \text{unconstr.}}^*}{d\bar{p}} = 1 - F_{(n-1)}(\bar{p}) > 0 . \quad (6.5)$$

(ii) Under constrained threshold entry (Corollary 5),

$$\frac{d\pi_{A, \text{constr.}}^*}{d\bar{p}} = 1 - F(\bar{p})^n + n f(\bar{p})(c - \bar{p} F(\bar{p})^{n-1}) > 1 - F_{(n-1)}(\bar{p}) > 0. \quad (6.6)$$

(iii) Under stochastic entry (Proposition 3), with optimal $b_0^* = 0$,

$$\frac{d\pi_{A, \text{stoch.}}^*}{d\bar{p}} = 1 - \left(\frac{c}{\bar{p}}\right)^{\frac{n}{n-1}} > 0. \quad (6.7)$$

Since π_A^* is continuous at the transitions between regimes, strict monotonicity holds globally.

Proof. Part (i) is derived in Appendix C by applying the Leibniz rule to (5.11); the expression $1 - F_{(n-1)}(\bar{p})$ is strictly positive for $\bar{p} < \bar{v}$.

For part (ii), substituting $b_0 = \bar{p} - c/F(\bar{p})^{n-1}$ from Corollary 5 into (5.17) yields $\pi_{A, \text{constr.}}^* = \bar{p}(1 - F(\bar{p})^n) - nc(1 - F(\bar{p}))$. Differentiating with respect to $\bar{p}$:

$$\frac{d\pi_{A, \text{constr.}}^*}{d\bar{p}} = 1 - F(\bar{p})^n - n\bar{p}f(\bar{p})F(\bar{p})^{n-1} + nc f(\bar{p}).$$

To establish the lower bound, note that Corollary 5 requires $c > \phi(\bar{p})F(\bar{p})^{n-1}$. Extending both sides by $nf(\bar{p})$ and explicitly writing the virtual value function:

$$nc f(\bar{p}) > n\bar{p}f(\bar{p})F(\bar{p})^{n-1} - n(1 - F(\bar{p}))F(\bar{p})^{n-1}.$$

Substituting:

$$\frac{d\pi_{A, \text{constr.}}^*}{d\bar{p}} > 1 - F(\bar{p})^n - n(1 - F(\bar{p}))F(\bar{p})^{n-1} = 1 - F_{(n-1)}(\bar{p}) > 0.$$

For part (iii), differentiating (5.17) at $b_0 = 0$ yields $1 - (c/\bar{p})^{n/(n-1)}$, which is strictly positive whenever $c < \bar{p}$ - the condition required for a viable auction. $\square$

Let $\Delta(c, \bar{p}) = \pi_A^* - \pi_P^*$ denote the profit advantage of the auction over the posted price. We first characterise this advantage at $c = 0$.

Proposition 8 (Level and gradient of Δ at $c = 0$). *At $c = 0$, the entry equilibrium is always of the deterministic threshold type and the unconstrained optimal reservation bid is*

$b_0^{**} = \tilde{v}^* = \phi^{-1}(0) \equiv b_0^{opt}$, the optimal reserve price in absence of any entry frictions. The unconstrained posted-price optimum is $p^{**} = \phi_{(n)}^{-1}(0)$.

(a) $\Delta(0, \bar{p}) > 0$ if and only if $\bar{p} > b_0^{opt}$. For $\bar{p} \leq b_0^{opt}$, both mechanisms yield equal revenue $\bar{p}(1 - F(\bar{p})^n)$.

(b) $\partial\Delta(0, \bar{p})/\partial\bar{p} > 0$ for all $\bar{p} > b_0^{opt}$: the auction advantage over the posted price erodes strictly as $\bar{p}$ declines.

Proof. We first establish $b_0^{opt} < p^{**}$, used in both parts. By Lemma 1(iii), $\lambda_{(n)}(v) < \lambda(v)$ for all v , which is equivalent to $\phi_{(n)}(v) < \phi(v)$. Evaluating at $v = b_0^{opt}$:

$$\phi_{(n)}(b_0^{opt}) < \phi(b_0^{opt}) = 0 = \phi_{(n)}(p^{**}) ,$$

where the equality $\phi(b_0^{opt}) = 0$ is the definition of b_0^{opt} and $\phi_{(n)}(p^{**}) = 0$ is the definition of p^{**} . Since $\phi_{(n)}$ is strictly increasing (Lemma 1(ii)), it follows that $b_0^{opt} < p^{**}$.

Part (a). *Case $\bar{p} \leq b_0^{opt}$:* Under both mechanisms, the unconstrained optima b_0^{opt} and p^{**} lie above $\bar{p}$. For the auction, any $b_0 > \bar{p}$ attracts no bidders (all bids are capped at $\bar{p}$ by Lemma 2), and profit is increasing in b_0 for $b_0 \leq \bar{p}$, so the constrained optimum is $b_0^* = \bar{p}$. At $c = 0$, every buyer with $v_i \geq \bar{p}$ enters and bids $\bar{p}$; the good sells at $\bar{p}$ whenever at least one such buyer exists, yielding $\pi_A^* = \bar{p}(1 - F(\bar{p})^n)$. The posted price is similarly constrained at $p^* = \bar{p}$, giving $\pi_P^* = \bar{p}(1 - F(\bar{p})^n)$. Hence $\Delta(0, \bar{p}) = 0$.

*Case $\bar{p} \in (b_0^{opt}, p^{**}]$:* The posted price is weakly constrained at $p^* = \bar{p}$, so $\pi_P^* = \bar{p}(1 - F(\bar{p})^n)$. At $b_0 = \bar{p}$, the auction generates the same revenue: every entrant bids $\bar{p}$, so the seller receives $\bar{p}$ whenever at least one buyer enters. Since $b_0^{opt} < \bar{p}$ is the strict interior maximizer of auction profit:

$$\pi_A^*(b_0^{opt}) > \pi_A(b_0 = \bar{p}) = \bar{p}(1 - F(\bar{p})^n) = \pi_P^* .$$

*Case $\bar{p} > p^{**}$:* The posted price is unconstrained. Setting $b_0 = p^{**}$ in the auction, competitive bidding above p^{**} generates additional revenue:

$$\pi_A(b_0 = p^{**}) = \pi_P(p^{**}) + \int_{p^{**}}^{\bar{p}} (v - p^{**}) f_{(n-1)}(v) dv + \int_{\bar{p}}^{\bar{v}} (\bar{p} - p^{**}) f_{(n-1)}(v) dv . \quad (6.8)$$

Both integrals are strictly positive since $p^{**} < \bar{p}$ and $n \geq 2$. Since b_0^{opt} is the unique maximizer:

$$\pi_A^*(b_0^{opt}) > \pi_A(b_0 = p^*) > \pi_P^* .$$

Part (b). *Case $\bar{p} > p^{**}$:* The posted price is unconstrained, so $\partial\pi_P^*/\partial\bar{p} = 0$, and $\partial\Delta/\partial\bar{p} = 1 - F_{(n-1)}(\bar{p}) > 0$ by Proposition 7(i).

*Case $\bar{p} \in (b_0^{\text{opt}}, p^{**}]$:* The posted price is constrained at $\bar{p}$, so $\pi_P^* = \bar{p}(1 - F(\bar{p})^n)$. Using the formula derived in Appendix D:

$$\frac{\partial\Delta}{\partial\bar{p}} = n F(\bar{p})^{n-1} f(\bar{p}) \underbrace{\left[\bar{p} - \frac{1 - F(\bar{p})}{f(\bar{p})} \right]}_{\phi(\bar{p})}.$$

Since ϕ is strictly increasing with $\phi(b_0^{\text{opt}}) = 0$, it follows that $\phi(\bar{p}) > 0$ for all $\bar{p} > b_0^{\text{opt}}$, and hence $\partial\Delta/\partial\bar{p} > 0$. $\square$

We now extend the gradient result to the case $c > 0$.

Proposition 9 (Gradient of the profit advantage). *For any $c > 0$, the gradient $\partial\Delta(c, \bar{p})/\partial\bar{p}$ is strictly positive for any type of threshold entry and therefore implicitly also strictly positive throughout the parameter region where $\pi_A^* \geq \pi_P^*$.*

Proof. Unconstrained threshold entry. For $\bar{p} > p^{**}$, the argument of Proposition 8(b), Case 1 applies without modification: $\partial\Delta/\partial\bar{p} = 1 - F_{(n-1)}(\bar{p}) > 0$.

For $\bar{p} \leq p^{**}$, the posted price is constrained and Appendix D gives:

$$\frac{\partial\Delta}{\partial\bar{p}} = n F(\bar{p})^{n-1} f(\bar{p}) \phi(\bar{p}).$$

Since $\partial\pi_{A,\text{unconstr.}}^*/\partial\bar{p}$ does not depend on c (Proposition 7(i)), this formula holds for all $c \geq 0$. In the unconstrained threshold entry regime, the entry threshold satisfies $\tilde{v}^*(c) \leq \bar{p}$ by definition. The FOC (5.13) gives $\phi(\tilde{v}^*(c)) = c/F(\tilde{v}^*(c))^{n-1} > 0$. Since ϕ is strictly increasing and $\bar{p} \geq \tilde{v}^*(c)$:

$$\phi(\bar{p}) \geq \phi(\tilde{v}^*(c)) > 0,$$

so $\partial\Delta/\partial\bar{p} > 0$.

Constrained threshold entry. Again, for $\bar{p} > p^{**}$, $\partial\pi_P^*/\partial\bar{p} = 0$ and therefore $\partial\Delta/\partial\bar{p} = \partial\pi_{A,\text{constr.}}^*/\partial\bar{p} > 0$ from Proposition 7(ii).

For $\bar{p} \leq p^{**}$, the posted price is constrained and direct subtraction of equation (6.4) from (6.6) yields:

$$\frac{\partial\Delta}{\partial\bar{p}} = n f(\bar{p}) c > 0.$$

Stochastic entry. By Proposition 5, the stochastic entry regime satisfies $\pi_A^* < \pi_P^*$, so it lies outside the parameter region of this proposition.

For completeness, we derive the gradient under stochastic entry, under the assumption of a constrained posted price. Subtraction of equation (6.4) from (6.7) yields:

$$\begin{aligned}\frac{\partial \Delta}{\partial \bar{p}} &= \left[1 - \left(\frac{c}{\bar{p}} \right)^{\frac{n}{n-1}} \right] - [1 - F(\bar{p})^n - n \bar{p} f(\bar{p}) F(\bar{p})^{n-1}] \\ &= F(\bar{p})^n + n \bar{p} f(\bar{p}) F(\bar{p})^{n-1} - \left(\frac{c}{\bar{p}} \right)^{\frac{n}{n-1}}.\end{aligned}$$

The sign of this expression is algebraically ambiguous and most likely to turn negative when c approaches $\bar{p}$, i.e. precisely where auction profit nears zero and the auction completely loses the ability to attract entrants. Intuitively, this makes sense because at this point, the posted price mechanism may still be viable and therefore still impacted by a further decrease in $\bar{p}$. $\square$

Proposition 10 (Unique crossing points). *(a) For any $\bar{p} > b_0^{opt}$, there exists a unique $\hat{c}(\bar{p}) > 0$ such that $\pi_A^*(\hat{c}, \bar{p}) = \pi_P^*(\bar{p})$. The seller strictly prefers the auction for $c < \hat{c}(\bar{p})$ and the posted price for $c > \hat{c}(\bar{p})$.*

(b) For any $c > 0$, there exists at most one $\hat{p}(c)$ such that $\pi_A^(c, \hat{p}) = \pi_P^*(\hat{p})$.*

Proof. Part (a): For any $\bar{p} > b_0^{opt}$, Proposition 8(a) gives $\Delta(0, \bar{p}) > 0$. By Proposition 6, Δ is continuous and strictly decreasing in c , with $\Delta(c, \bar{p}) \rightarrow -\pi_P^*(\bar{p}) < 0$ as $c \rightarrow \min\{\bar{p}, \bar{v}\}$. The intermediate value theorem yields existence of $\hat{c}(\bar{p})$; strict monotonicity of Δ in c ensures uniqueness.

Part (b): By Proposition 9, $\partial \Delta / \partial \bar{p} > 0$ throughout the region $\pi_A^* \geq \pi_P^*$. Hence Δ is strictly increasing in $\bar{p}$ while non-negative, so it can cross zero from above at most once as $\bar{p}$ decreases. $\square$

6.4 Mechanism-Switching Boundary

The preceding results jointly characterise the seller's preferred mechanism across the entire $(c, \bar{p})$ parameter space. Define $\hat{c}(\bar{p})$ from Proposition 10 as the *switching threshold*: the entry cost at which the seller is indifferent between the two mechanisms, given outside option price $\bar{p}$.

Proposition 11 (Mechanism-switching boundary).

- (i) For any $\bar{p} > b_0^{\text{opt}}$, the switching threshold $\hat{c}(\bar{p}) > 0$ is uniquely defined by $\Delta(\hat{c}(\bar{p}), \bar{p}) = 0$, and is strictly increasing in $\bar{p}$:

$$\frac{d\hat{c}}{d\bar{p}} = -\frac{\partial\Delta/\partial\bar{p}}{\partial\Delta/\partial c} > 0. \quad (6.9)$$

A less attractive outside option therefore tolerates a larger entry cost before the posted price becomes preferable.

- (ii) For $\bar{p} \leq b_0^{\text{opt}}$, the posted price weakly dominates the auction for all $c \geq 0$, with strict dominance whenever $c > 0$.

Proof. Part (i): Existence and uniqueness of $\hat{c}(\bar{p})$ for each $\bar{p} > b_0^{\text{opt}}$ follow from Proposition 10(a). The implicit function theorem applies at any point $(\hat{c}, \bar{p})$ on the boundary: $\partial\Delta/\partial c < 0$ by Proposition 6, and $\partial\Delta/\partial\bar{p} > 0$ by Proposition 9 (which holds in particular at $\Delta = 0$). Hence (6.9) is strictly positive.

Part (ii): By Proposition 8(a), $\Delta(0, \bar{p}) = 0$ for $\bar{p} \leq b_0^{\text{opt}}$. Strict decrease of Δ in c (Proposition 6) then gives $\Delta(c, \bar{p}) < 0$ for all $c > 0$. $\square$

Figure 4 illustrates the switching boundary $\hat{c}(\bar{p})$ in the $(c, \bar{p})$ -space for the numerical example. The auction-preferred region occupies the upper-left corner (low entry cost, high outside option price); the posted-price region expands as c rises or $\bar{p}$ falls. The boundary $\hat{c}(\bar{p})$ is upward-sloping, consistent with part (i): platforms on which outside options are less accessible can sustain auction dominance against higher participation frictions.

7 Discussion

This paper develops a second-price auction model that incorporates two forms of buyer opportunity cost: an entry cost c that is sunk upon participation, and a value-dependent outside option $w_i = \max\{v_i - \bar{p}, 0\}$ that represents the surplus a buyer can obtain by acquiring the good elsewhere. These two mechanisms admit a unified interpretation as *nested opportunity costs*: the entry cost captures the opportunity cost of *participating* in the auction - the monitoring effort, attention, and time that a buyer commits regardless of the outcome - while the outside option captures the opportunity cost of *winning* the auction, namely the forgone surplus from the next-best alternative. The two costs also differ in an important aspect: the entry cost is a friction exclusive to the auction format, borne only by

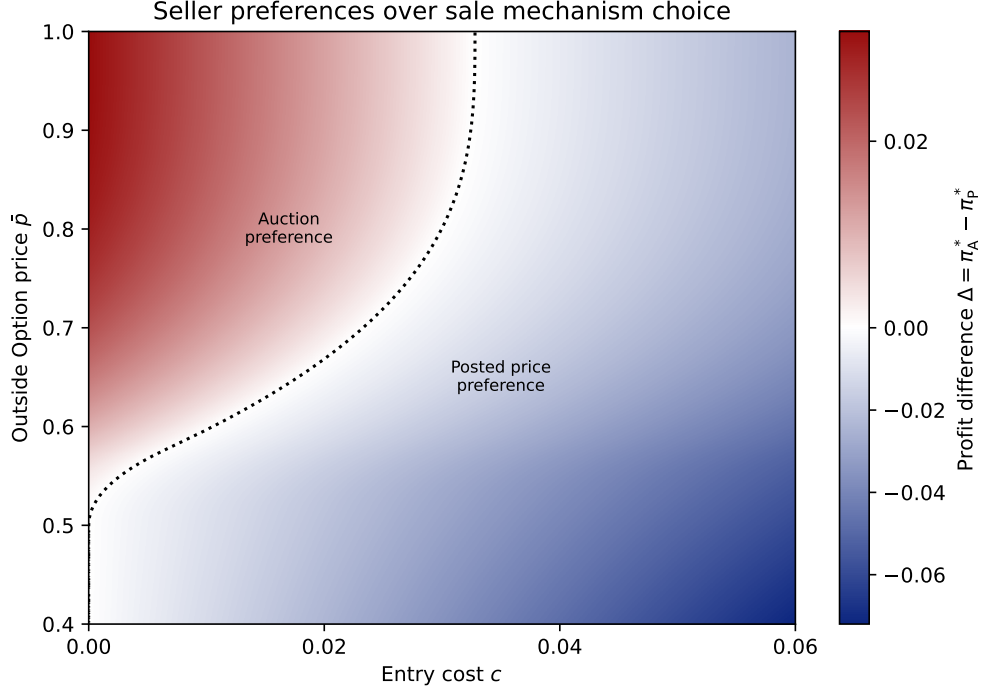


Figure 4: Auction profit advantage $\Delta(c, \bar{p})$ over the $(c, \bar{p})$ -space in the numerical example. The switching boundary $\hat{c}(\bar{p})$ is the zero contour and marked by the dotted line.

buyers who choose to participate and sunk upon participation, while the outside option is a feature of the broader market environment that constrains the seller's revenue under any pricing mechanism. As discussed in Sections 1 and 3, both types of opportunity cost are well-motivated in the context of digital marketplaces, where auction participation requires sustained engagement and close substitutes are increasingly accessible.

Structural Insights

The introduction of the outside option generates several structural features that go beyond the standard entry cost framework. The single parameter $\bar{p}$ creates an endogenous partition of buyers into two types within an otherwise symmetric model: low-value buyers ($v_i < \bar{p}$), who have no viable outside option and therefore bid their full valuation, and high-value buyers ($v_i \geq \bar{p}$), who are guaranteed to acquire the good from one source or another and whose bids are truncated at $\bar{p}$. This partition is the driver of the model's equilibrium structure. Because high-value buyers are symmetric in their net valuation of the auctioned good - each bids exactly $\bar{p}$, regardless of their private value - the model gives rise to two qualitatively distinct entry regimes (Proposition 1). Under deterministic threshold entry, both low- and high-value buyers may participate, and the entry margin is governed by a critical valuation

$\tilde{v}$ as in the standard framework of Samuelson [1985]. Under stochastic entry, all low-value buyers are priced out, and high-value buyers randomize over participation in a manner that is structurally analogous to ex ante symmetric entry models such as McAfee and McMillan [1987] and Levin and Smith [1994]: because high-value buyers are symmetric in their effective valuation, no deterministic selection among them is possible.

The seller's optimization problem under unconstrained threshold entry yields the extended virtual value function $\phi_{\text{entry}}(\tilde{v}) = \phi(\tilde{v}) - c/F(\tilde{v})^{n-1}$ (Corollary 4). This decomposition transparently separates two rents that the seller must concede: the standard information rent $\tilde{v} - \phi(\tilde{v})$ of Myerson [1981], which compensates buyers for truthful revelation of their valuations, and an *entry rent* $c/F(\tilde{v})^{n-1}$, which compensates the marginal entrant for the sunk participation cost. The entry rent is decreasing in $F(\tilde{v})$ and therefore in the number of buyers below the threshold, reflecting that entry is less costly to sustain when fewer competitors must be attracted. This extended virtual value nests Myerson's standard result as the special case $c = 0$.

A further noteworthy structural result concerns the role of the optimal frictionless reservation bid $b_0^{\text{opt}} = \phi^{-1}(0)$ as a critical threshold for $\bar{p}$. Proposition 8(a) and Proposition 11(ii) jointly establish that whenever $\bar{p} \leq b_0^{\text{opt}}$, the posted price weakly dominates the auction for *any* level of entry costs, including $c = 0$. The outside option alone, without any participation friction, can eliminate the auction's revenue advantage. The intuition is that when the outside option price falls at or below the optimal frictionless reservation bid, competition among bidders is capped precisely in the valuation range where the auction would otherwise generate its surplus over posted prices. The competitive bidding that constitutes the auction's principal advantage over the posted price is neutralized by the ceiling that $\bar{p}$ imposes on equilibrium bids.

Seller Behavior and the Asymmetry of Responses

While the entry equilibrium from the buyers' perspective takes only two forms - deterministic threshold or stochastic (Proposition 1) - the seller's optimal reservation bid distinguishes *three* regimes. Under unconstrained threshold entry, the interior first-order condition is feasible and both low- and high-value buyers may participate (Proposition 2). Under constrained threshold entry, the entry cost is too high for the unconstrained optimum to lie within the feasible range, and the seller lowers b_0 to the regime boundary, placing the entry threshold at $\tilde{v} = \bar{p}$: all low-value buyers are priced out, but every high-value buyer still participates with certainty (Corollary 5). Under stochastic entry, no non-negative reservation bid can sustain deterministic entry, and the seller optimally sets $b_0^* = 0$ (Proposition 3). The seller

therefore uses the reservation bid as a tool to *avoid* the stochastic entry regime - lowering b_0 to pull the equilibrium back toward deterministic threshold entry. Notably, once the seller is pushed out of the unconstrained regime, the transition is sharp: the seller sacrifices all low-value buyer participation but clings to full high-value buyer entry for as long as this can be sustained.

Both an increase in the entry cost c and a decrease in the outside option price $\bar{p}$ erode the auction's profitability: the comparative statics in Propositions 6 and 7 show that auction profit is strictly decreasing in c and strictly increasing in $\bar{p}$. In both cases, the seller seeks to counteract falling revenue by reducing the reservation bid. However, the *timing* of this policy response differs markedly across the two channels.

When the entry cost increases, the seller responds immediately by adjusting the reservation bid downward (Lemma 3). The optimal b_0^{**} under unconstrained threshold entry depends on c through the entry condition (5.3): a higher c raises the entry threshold $\tilde{v}$, and the seller lowers b_0 to partially offset the entry deterrence. This adjustment occurs smoothly.

By contrast, a decline in $\bar{p}$ does not trigger an adjustment of b_0^{**} for as long as the auction remains in the unconstrained threshold regime. This is because the optimal reservation bid (5.14) depends only on the entry threshold $\tilde{v}$, which in turn is determined by c and b_0 via the entry condition but not by $\bar{p}$ directly. The outside option price affects auction profit by compressing the ceiling on competitive bids (Proposition 7(i)), but this damage passes through to the seller without activating any compensating instrument. Only once $\bar{p}$ falls far enough that the seller is forced out of the interior optimum - into the constrained threshold regime or stochastic entry - does b_0 adjust downward. At that point, however, the auction has already deteriorated past the switching boundary $\hat{c}(\bar{p})$, i.e. the posted price is already the more profitable mechanism (Proposition 5). In short, the reservation bid responds to entry costs proactively but to outside option pressure only reactively, and by the time the reaction occurs, the auction's advantage against the posted price has already vanished.

Implications for E-Commerce

The model provides a two-channel explanation for the secular shift from auctions to posted prices on digital marketplaces, one that operates through fully rational, risk-neutral buyers and does not rely on behavioral biases. The first channel runs through the outside option: as search technology improves and product variety expands, the effective price $\bar{p}$ at which a buyer can acquire a close substitute declines. The second channel runs through the entry cost: as platforms streamline purchase processes and shorten delivery times, the relative

hassle of auction participation increases. As noted in Section 1, there is considerably stronger evidence for a secular improvement in buyers’ outside options - driven by falling search costs, expanding product variety, and increasingly sophisticated recommendation algorithms - than for a clear directional change in auction entry costs. This asymmetry of evidence places the outside option channel, which is the more novel theoretical mechanism of this paper, at the center of scrutiny.

These two channels are empirically distinct and potentially separable. The comparative statics established in Section 6 show that each channel independently erodes the auction’s revenue advantage over the posted price, and that their effects compound: the switching boundary $\hat{c}(\bar{p})$ is upward-sloping (Proposition 11(i)), so a platform experiencing both falling $\bar{p}$ and rising c faces a doubly contracting auction-preferred region.

The model further generates predictions about buyer behavior and seller responses under changes in both channels. A fully-fledged parameter estimation is outside the scope of this paper, but the rich dataset of Einav et al. (2018) provides several descriptive findings that lend themselves to a qualitative comparison with the model’s predictions. We examine four of their observations and assess which channel - outside option or entry cost - is more consistent with each.

1. Cross-category coexistence of mechanisms. Einav et al. (2018) document that auctions on eBay are substantially more common for used goods and for items with fewer duplicate listings - precisely the categories where close substitutes are scarce. Since the parameters c and $\bar{p}$ vary across product categories, the switching boundary $\hat{c}(\bar{p})$ implies that auctions should persist for goods with few close substitutes (high $\bar{p}$), while posted prices should dominate for standardized goods with many available alternatives (low $\bar{p}$). This is closely aligned with the outside option channel: categories in which outside options are abundant shifted to posted prices, while idiosyncratic goods for which outside options are limited retained the auction format. The entry cost channel offers no comparably sharp cross-category prediction, since monitoring and participation costs are not assumed to vary as systematically across product types.

2. Reserve price inertia despite a growing auction discount. Over their sample period, Einav et al. (2018) find that the auction price discount relative to matched posted-price listings grew substantially, yet sellers did not offset this erosion by adjusting reserve prices downward; in fact, reserve prices shifted slightly upward in relative terms. The model generates distinct predictions for each channel. Under entry cost pressure, the seller proactively

lowers the reservation bid while remaining in the auction-preferred region (Lemma 3): one would therefore expect a period of declining starting bids preceding the format switch. Under outside option pressure, by contrast, the optimal reservation bid does not adjust for as long as the auction remains in the unconstrained threshold regime (Proposition 5), because b_0^{**} depends on the entry threshold $\tilde{v}$, which is determined by c and b_0 but not by $\bar{p}$ directly. No defensive reserve price reduction should therefore be observed before sellers abandon the format. The empirical pattern favors the latter: the auction discount widened without a compensating seller response, which is precisely the signature the model predicts when the outside option, rather than the entry cost, is the dominant channel. The slight relative increase in reservation bids can be reconciled with the model if the optimal posted price decreases faster than the optimal reservation bid under the relevant pressures, a possibility examined in point 4 below.

3. Stable relative sale probability despite a growing auction discount. Einav et al. [2018] report that the auction discount increased over time, while the relative difference in sale probability between the two formats remained relatively stable. Under entry cost pressure, the seller partially compensates the loss in entrants by lowering b_0 (Lemma 3), but this offset is only partial: $\tilde{v}$ still rises, reducing entry and therefore the auction’s sale probability relative to the posted price. Under outside option pressure in the unconstrained threshold regime, by contrast, the entry threshold $\tilde{v}$ is independent of $\bar{p}$, so entry - and hence the auction’s sale probability - remains unchanged even as the bid ceiling $\bar{p}$ compresses the expected winning price. The growing auction discount would therefore reflect the mechanical reduction in high-value buyers’ bids rather than a reduction in participation. This combination - a widening discount with stable relative sale probability - is naturally predicted by the outside option channel but difficult to reconcile with the entry cost channel, which would imply a simultaneous deterioration of the auction’s sale probability.

4. Declining sale probability and bidder count in both formats. Einav et al. [2018] also find that the average sale probability of both formats decreased significantly over their sample period, and that the average number of unique bidders in the auction fell. They interpret the latter effect, together with the growing auction discount from point 3, as evidence for a decline in relative auction demand and attribute it to the entry hassle channel. In this model, however neither of the two focal parameters offers a fully satisfactory explanation for this pattern. A decline in $\bar{p}$ does not affect entry in the unconstrained threshold regime, and an increase in c reduces auction entry but should be partially offset by a lower reservation bid; moreover, neither channel accounts for the reduction in successful posted-price sales. A

channel that would be largely consistent with this observation - although not a focal parameter of this paper - is a reduction in the number of potential buyers n . This channel is not considered in Einav et al. [2018] because buyers have a uniform valuation in their model, so variation in n does not affect the equilibrium. In the present model however, a reduction in n lowers sale probability in both formats under most circumstances, and therefore also reduces equilibrium entry in the auction. Furthermore, a reduction in n triggers a decline in the unconstrained optimal posted price, but affects the optimal reservation bid only through the entry rent (cf. Corollary 4), where $\partial b_0^{**}/\partial n$ is proportional to c . If c is not too large, this asymmetric sensitivity could explain the relative stability - or even relative increase - of b_0^{**} compared to the posted price observed by Einav et al. [2018]. A brief characterization of comparative statics in n is provided in Appendix E. An interpretation of a reduction in n can take several forms, for instance decreased visibility of individual listings due to increased competition on the platform, although the effect of competition in this model is also channeled through the outside option. Taking n as a focal parameter therefore raises questions of endogeneity between the parameters, which is why the main analysis abstains from this direction.

While these comparisons are only qualitative, the model's predictions are largely consistent with the empirical patterns documented by Einav et al. [2018]. Across the four observations, the outside option channel provides the more coherent account: it explains cross-category variation (point 1), the absence of defensive seller responses (point 2), and the stability of relative sale probabilities (point 3), while the entry cost appears to play a subordinate role. The residual observation of declining absolute sale probabilities and bidder counts (point 4) may point towards a complementary channel - variation in n - that warrants further investigation. A structural estimation exercise could help assess the quantitative explanatory power of the model and disentangle the relative contributions of the two channels.

Welfare

The analysis in this paper is conducted entirely from the perspective of seller revenue. A brief qualitative observation on welfare is nonetheless warranted. The auction's principal welfare advantage over posted prices rests on its allocation property: a second-price auction assigns the good to the highest-valued participant, whereas a posted price, when multiple buyers are willing to purchase, does not guarantee this efficient allocation. Entry costs work against the auction's welfare standing: they consume real resources - time, attention, cognitive effort - without creating surplus, constituting a deadweight loss of magnitude $n \cdot c \cdot E[\delta_i]$ in equilibrium.

An increasingly attractive outside option erodes the allocation advantage. As $\bar{p}$ falls, a growing share of participants - all those with $v_i \geq \bar{p}$ - submit the common bid $\bar{p}$, and the auction loses its ability to discriminate among them. In the constrained threshold and stochastic entry regimes, this erosion is complete: only high-value buyers participate, all bid $\bar{p}$, and the winner is selected at random among entrants - the same allocation that a posted price at $\bar{p}$ would produce. Moreover, the indifference condition that characterizes entry in these regimes implies that buyers earn zero expected surplus relative to their outside option, so that total welfare coincides with seller profit. Under constrained threshold entry, where all high-value buyers enter with certainty, this yields

$$W_A = \bar{p}(1 - F(\bar{p})^n) - n c(1 - F(\bar{p})) ,$$

while the corresponding posted-price welfare at $p = \bar{p}$ is $W_P = \bar{p}(1 - F(\bar{p})^n)$: the same gross surplus, without the entry cost term. For any $c > 0$, the auction is therefore strictly welfare-inferior once the regime transitions past unconstrained threshold entry. Under stochastic entry, the comparison is even more stark: entry probability falls below one, so the auction generates strictly less gross surplus *and* still incurs participation costs. This welfare ranking aligns with the seller's own incentive: Proposition 5 establishes that the seller strictly prefers the posted price throughout these same regimes.

In the unconstrained threshold regime, the welfare comparison is ambiguous in general. The auction retains a genuine allocation advantage among low-value bidders, who bid true to their full valuation and are therefore sorted efficiently, but this gain must be weighed against the deadweight loss from entry costs.

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A Derivative of Seller Profit under Deterministic Threshold Entry

We differentiate the profit function (5.11) with respect to $\tilde{v}$:

$$\begin{aligned}\pi_{A, \text{det.}} &= \left(\tilde{v} - \frac{c}{F(\tilde{v})^{n-1}} \right) \binom{n}{1} F(\tilde{v})^{n-1} (1 - F(\tilde{v})) + \int_{\tilde{v}}^{\bar{p}} v f_{(n-1)}(v) dv + \int_{\bar{p}}^{\tilde{v}} \bar{p} f_{(n-1)}(v) dv \\ &= n(1 - F(\tilde{v}))(\tilde{v} F(\tilde{v})^{n-1} - c) + \int_{\tilde{v}}^{\bar{p}} v f_{(n-1)}(v) dv + \int_{\bar{p}}^{\tilde{v}} \bar{p} f_{(n-1)}(v) dv.\end{aligned}\quad (\text{A.1})$$

Applying the product rule to the first term and the Leibniz integral rule to the second yields:

$$\begin{aligned}\frac{\partial \pi_{A, \text{det.}}}{\partial \tilde{v}} &= -n f(\tilde{v})(\tilde{v} F(\tilde{v})^{n-1} - c) \\ &\quad + n(1 - F(\tilde{v}))[F(\tilde{v})^{n-1} + (n-1)\tilde{v} f(\tilde{v}) F(\tilde{v})^{n-2}] \\ &\quad - \tilde{v} f_{(n-1)}(\tilde{v}).\end{aligned}\quad (\text{A.2})$$

Substituting the density of the $(n-1)$ -th order statistic, $f_{(n-1)}(\tilde{v}) = n(n-1) f(\tilde{v}) F(\tilde{v})^{n-2} (1 - F(\tilde{v}))$, the third term cancels the second term in bracket of the second line, leaving:

$$\begin{aligned}\frac{\partial \pi_{A, \text{det.}}}{\partial \tilde{v}} &= -n f(\tilde{v})(\tilde{v} F(\tilde{v})^{n-1} - c) + n(1 - F(\tilde{v})) F(\tilde{v})^{n-1} \\ &= n \left[F(\tilde{v})^{n-1} (1 - F(\tilde{v}) - \tilde{v} f(\tilde{v})) + c f(\tilde{v}) \right].\end{aligned}\quad (\text{A.3})$$

Setting (A.3) equal to zero, dividing by $n f(\tilde{v}) > 0$, and rearranging yields the first-order condition:

$$\frac{c}{F(\tilde{v})^{n-1}} = \tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})}.\quad (\text{A.4})$$

B Formation and Differentiation of Seller Profit under Stochastic Entry

As per equation [5.16](#), the sellers profit function under stochastic entry can be conceptually stated as:

$$\pi_{A, \text{stoch.}} = \bar{p} P(k \geq 1) - n c \tilde{\delta}_H (1 - F(\bar{p})). \quad (\text{B.1})$$

Each agent enters with probability $\tilde{\delta}_H (1 - F(\bar{p}))$. The probability that at least one of the n agents enters is:

$$P(k \geq 1) = 1 - P(k = 0) = 1 - (1 - \tilde{\delta}_H (1 - F(\bar{p})))^n,$$

The independent entry probability $\tilde{\delta}_H$ of a high-value buyer is given by Proposition [1](#):

$$\tilde{\delta}_H = \frac{1 - \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{1}{n-1}}}{1 - F(\bar{p})}$$

Substitution and simplification:

$$\begin{aligned} \pi_{A, \text{stoch.}} &= \bar{p} \left(1 - \left(1 - \frac{1 - \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{1}{n-1}}}{1 - F(\bar{p})} \cdot (1 - F(\bar{p})) \right)^n \right) - n c \cdot \frac{1 - \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{1}{n-1}}}{1 - F(\bar{p})} \cdot (1 - F(\bar{p})) \\ &= \bar{p} \left(1 - \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{n}{n-1}} \right) - n c \left(1 - \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{1}{n-1}} \right). \end{aligned} \quad (\text{B.2})$$

Differentiating [\(B.2\)](#) with respect to b_0 :

$$\begin{aligned} \frac{\partial \pi_{A, \text{stoch.}}}{\partial b_0} &= -\bar{p} c^{\frac{n}{n-1}} \cdot \frac{n}{n-1} \cdot \left(\frac{1}{\bar{p} - b_0}\right)^{\frac{2n-1}{n-1}} + c^{\frac{n}{n-1}} \cdot \frac{n}{n-1} \cdot \left(\frac{1}{\bar{p} - b_0}\right)^{\frac{n}{n-1}} \\ &= c^{\frac{n}{n-1}} \cdot \frac{n}{n-1} \cdot \left(\left(\frac{1}{\bar{p} - b_0}\right)^{\frac{n}{n-1}} - \bar{p} \left(\frac{1}{\bar{p} - b_0}\right)^{\frac{2n-1}{n-1}} \right) \\ &= \frac{n}{n-1} \cdot \left(\frac{c}{\bar{p} - b_0}\right)^{\frac{n}{n-1}} \cdot \frac{-b_0}{\bar{p} - b_0}. \end{aligned} \quad (\text{B.3})$$

For $b_0 > 0$, every factor except $-b_0$ is strictly positive, so $\partial \pi_{A, \text{stoch.}} / \partial b_0 < 0$. At $b_0 = 0$, the derivative equals zero. The seller therefore optimally sets $b_0^* = 0$.

C Derivative of seller profit under unconstrained threshold entry, with respect to $\bar{p}$

We differentiate the following profit function with respect to $\bar{p}$:

$$\pi = b_0(\tilde{v}) \cdot \binom{n}{1} \cdot F(\tilde{v})^{n-1} \cdot (1 - F(\tilde{v})) + \int_{\tilde{v}}^{\bar{p}} v \cdot f_{(n-1)}(v) dv + \int_{\bar{p}}^{\bar{v}} \bar{p} \cdot f_{(n-1)}(v) dv$$

The first term is constant in $\bar{p}$ and therefore falls away. The two following integrals can be differentiated by application of the Leibniz integral rule, to yield:

$$\begin{aligned} \frac{\partial \pi}{\partial \bar{p}} &= \bar{p} \cdot f_{(n-1)}(\bar{p}) + \int_{\bar{p}}^{\bar{v}} f_{(n-1)}(v) dv - \bar{p} \cdot f_{(n-1)}(\bar{p}) \\ &= \int_{\bar{p}}^{\bar{v}} f_{(n-1)}(v) dv \\ &= 1 - F_{(n-1)}(\bar{p}) \end{aligned}$$

D Gradient Comparison: Auction vs. Posted Price

We derive the expression for $\partial\Delta/\partial\bar{p}$ under unconstrained threshold entry.

Posted price. When $\bar{p} \leq p^{**}$, the outside option binds and $\pi_P^* = \bar{p}(1 - F(\bar{p})^n)$, so:

$$\frac{\partial\pi_P^*}{\partial\bar{p}} = 1 - F(\bar{p})^n - n\bar{p}f(\bar{p})F(\bar{p})^{n-1}.$$

For $\bar{p} > p^{**}$ the posted price is unconstrained and $\partial\pi_P^*/\partial\bar{p} = 0$.

Auction – Unconstrained Threshold Entry. From Proposition 7(i), using $F_{(n-1)}(\bar{p}) = nF(\bar{p})^{n-1} - (n-1)F(\bar{p})^n$:

$$\frac{\partial\pi_{A,\text{unconstr.}}^*}{\partial\bar{p}} = 1 - nF(\bar{p})^{n-1} + (n-1)F(\bar{p})^n.$$

Difference – Unconstrained Threshold Entry. For $\bar{p} \geq p^{**}$: $\partial\Delta/\partial\bar{p} = 1 - F_{(n-1)}(\bar{p}) > 0$.

For $\bar{p} < p^{**}$, writing $F = F(\bar{p})$ and $f = f(\bar{p})$ for brevity:

$$\begin{aligned} \frac{\partial\Delta}{\partial\bar{p}} &= [1 - nF^{n-1} + (n-1)F^n] - [1 - F^n - n\bar{p}fF^{n-1}] \\ &= -nF^{n-1} + nF^n + n\bar{p}fF^{n-1} \\ &= nF^{n-1}[F - 1 + \bar{p}f] \\ &= nF(\bar{p})^{n-1}f(\bar{p}) \underbrace{\left[\bar{p} - \frac{1 - F(\bar{p})}{f(\bar{p})}\right]}_{\phi(\bar{p})} \end{aligned}$$

Since $\partial\pi_{A,\text{unconstr.}}^*/\partial\bar{p}$ is independent of c , this formula holds for all $c \geq 0$. The sign of $\partial\Delta/\partial\bar{p}$ equals the sign of $\phi(\bar{p})$.

E Comparative Statics in the Number of Buyers n

This appendix derives how the optimal seller instruments and the sale probability respond to changes in the number of potential buyers n , under both the auction and the posted price. Transitions between cases are omitted.

Auction

Unconstrained threshold entry

Writing the first-order condition (5.13) as

$$G(\tilde{v}^*, n) = \phi(\tilde{v}^*) - \frac{c}{F(\tilde{v}^*)^{n-1}} = 0,$$

implicit differentiation yields

$$\frac{d\tilde{v}^*}{dn} = \frac{\frac{c}{F(\tilde{v}^*)^{n-1}} \ln(1/F(\tilde{v}^*))}{\phi'(\tilde{v}^*) + \frac{c(n-1)f(\tilde{v}^*)}{F(\tilde{v}^*)^n}} > 0.$$

Since $b_0^{**} = 1/\lambda(\tilde{v}^*)$ by (5.14) and the hazard rate λ is strictly increasing (Lemma 1(i)), the chain rule gives

$$\frac{db_0^{**}}{dn} = \frac{\partial b_0^{**}}{\partial \tilde{v}^*} \cdot \frac{d\tilde{v}^*}{dn} < 0.$$

In their magnitude, both effects are dependent on c and vanish as $c \rightarrow 0$.

For the sales probability $S = 1 - F(\tilde{v}^*)^n$, let $\tilde{v}$ denote the entry threshold that keeps S constant, defined implicitly by $\tilde{G}(\tilde{v}, n) = 1 - F(\tilde{v})^n = \text{const}$. Implicit differentiation yields

$$\frac{\partial \tilde{v}}{\partial n} = \frac{F(\tilde{v})^n \ln(1/F(\tilde{v}))}{n f(\tilde{v}) F(\tilde{v})^{n-1}} = \frac{F(\tilde{v}) \ln(1/F(\tilde{v}))}{n f(\tilde{v})}.$$

The optimal sale probability increases in n if and only if

$$\frac{\partial \tilde{v}}{\partial n} > \frac{d\tilde{v}^*}{dn},$$

i.e. the optimizing seller raises the threshold by less than would be needed to hold demand fixed. Direct comparison and rearrangement reduces this to

$$\frac{F(\tilde{v}^*)}{f(\tilde{v}^*)} \phi'(\tilde{v}^*) > \phi(\tilde{v}^*).$$

Although this reduced condition does not explicitly involve c or n , these parameters enter implicitly through condition 5.13 by determining the threshold $\tilde{v}^*$ at which it is evaluated. At $c = 0$, the threshold coincides with the optimal frictionless reserve value, where $\phi(\tilde{v}^*) = 0$, and the condition holds trivially. As c or n increase, $\tilde{v}^*$ is pushed upward into a region where

$\phi(\tilde{v}^*)$ is strictly positive and growing, so that the condition becomes increasingly demanding. Whether it continues to hold depends on the specific distribution: for distributions whose density is non-decreasing or only mildly concentrated toward the upper end of the support - such as the uniform, truncated exponential, or Beta(a, b) with $b \geq a$ - the condition is satisfied for all feasible parameter combinations. For distributions with density strongly concentrated near the upper bound - such as power distributions $F(v) = v^\alpha$ with $\alpha \geq 3$ or Beta(a, b) with $a \gg b$ - the elasticity can fall below unity, and the sale probability may decrease in n . Numerically, however, such violations typically only arise in a narrow region near the viability boundary, suggesting that the condition holds across the bulk of the feasible parameter space.

Constrained threshold entry

By Corollary 5, under constrained threshold entry the reservation bid is $b_0^* = \bar{p} - c/F(\bar{p})^{n-1}$, so

$$\frac{db_0^*}{dn} = -\frac{c}{F(\bar{p})^{n-1}} \ln(1/F(\bar{p})) < 0.$$

The magnitude of the response of b_0 to a change in n is proportional to c . The sales probability equals the probability of at least one high-value buyer, $S = 1 - F(\bar{p})^n$, and is strictly increasing in n :

$$\frac{dS}{dn} = F(\bar{p})^n \ln(1/F(\bar{p})) > 0.$$

Stochastic entry

This case is empirically of limited relevance but is included for completeness. By Proposition 3, $b_0^* = 0$ and is therefore invariant in n . The sales-probability response is even stronger than in the previous case because b_0 cannot be used to partially compensate. The sales probability is contingent on the probability of at least one high-value buyer, $1 - F(\bar{p})^n$, and the stochastic-equilibrium entry probability $\tilde{\delta}_H$ (cf. Proposition 1); both are increasing in n :

$$\frac{d(1 - F(\bar{p})^n)}{dn} = F(\bar{p})^n \ln(1/F(\bar{p})) > 0, \quad \frac{d\tilde{\delta}_H}{dn} = \frac{-(c/\bar{p})^{1/(n-1)} \ln(c/\bar{p})}{1 - F(\bar{p})} > 0.$$

Posted Price

Constrained posted price

In the constrained case the optimal price is invariant in n and given by $p^* = \bar{p}$. The sales probability $S = 1 - F(\bar{p})^n$ is strictly increasing in n :

$$\frac{dS}{dn} = F(\bar{p})^n \ln(1/F(\bar{p})) > 0.$$

Unconstrained posted price

Write $p = p^{**}$ for brevity. By the first-order condition (4.2), the optimal posted price satisfies $p = \gamma_{(n)}(p)$, where $\gamma_{(n)}$ is the reciprocal hazard rate of $F_{(n)}$. Using the decomposition established in the proof of Lemma 1(ii),

$$p = \gamma(p) A(p, n), \quad A(p, n) = \frac{1}{n} \sum_{j=0}^{n-1} F(p)^{-j},$$

where $\gamma(p) = (1 - F(p))/f(p)$ is the reciprocal hazard rate of F . This equation implicitly defines p as a function of n . With the first order condition $\phi_{(n)}(p, n) = p - \gamma(p) A(p, n) = 0$ and applying the implicit function theorem yields

$$\frac{dp}{dn} = -\frac{\partial \phi_{(n)}/\partial n}{\partial \phi_{(n)}/\partial p} = \frac{\gamma(p) \frac{\partial A}{\partial n}}{1 - \frac{\partial \gamma_{(n)}}{\partial p}}.$$

Signing each component:

- $\gamma(p) > 0$, since the reciprocal hazard rate is strictly positive on the interior of the support.
- $\partial A/\partial n > 0$: the factor $A(p, n)$ is the arithmetic mean of the strictly increasing sequence $1, F(p)^{-1}, \dots, F(p)^{-(n-1)}$. Adding the next term $F(p)^{-n}$, which exceeds the current mean, raises the average.
- $1 - \partial \gamma_{(n)}/\partial p > 1$: by Lemma 1(ii), $\partial \gamma_{(n)}/\partial p < 0$.

All three factors are strictly positive, so $dp^{**}/dn > 0$: *the optimal posted price is strictly increasing in the number of buyers.*

To sign the change in the sale probability, we compare the actual optimal price response dp^{**}/dn with the counterfactual response $\partial \tilde{p}/\partial n$ that keeps the sale probability $S = 1 - F(p)^n$ constant in n . The optimal sale probability is strictly increasing in n if and only if

$$\frac{\partial \tilde{p}}{\partial n} > \frac{dp^{**}}{dn},$$

i.e., the optimizing seller raises the price by less than what would be needed to offset the mechanical demand increase. Write $F = F(p)$, $f = f(p)$ for brevity and repeatedly invoke the FOC relation $S = 1 - F^n = npfF^{n-1}$.

Counterfactual response. Implicit differentiation of $\tilde{G}(p, n) = 1 - F^n = \text{const.}$ gives

$$\frac{\partial \tilde{p}}{\partial n} = \frac{F \ln(1/F)}{nf} = \frac{p F^n \ln(1/F)}{S}.$$

Optimal response. Setting $\phi_{(n)}(p, n) = p - \gamma_{(n)}(p, n)$ with $\gamma_{(n)} = S/(nfF^{n-1})$, the implicit function theorem delivers $dp^{**}/dn = -(\partial\phi_{(n)}/\partial n)/(\partial\phi_{(n)}/\partial p)$. Differentiating and simplifying via the FOC yields

$$\frac{\partial\phi_{(n)}}{\partial n} = \frac{p}{n} - \frac{p \ln(1/F)}{S},$$

and differentiating $\gamma_{(n)}$ in p together with $nfF^{n-1} = S/p$ gives

$$\frac{\partial\phi_{(n)}}{\partial p} = 2 + p \frac{f'}{f} + (n-1) \frac{pf}{F} > 1,$$

where the strict lower bound is by Lemma [1\(ii\)](#).

Reducing the criterion. Substituting the three derivatives into the criterion and multiplying through by $\partial\phi_{(n)}/\partial p > 0$ and $nS/p > 0$ yields

$$\frac{\partial\phi_{(n)}}{\partial p} \cdot nF^n \ln(1/F) > n \ln(1/F) - S.$$

Subtracting $nF^n \ln(1/F)$ from both sides and using $S = 1 - F^n$:

$$\left(\frac{\partial\phi_{(n)}}{\partial p} - 1 \right) nF^n \ln(1/F) > S(n \ln(1/F) - 1).$$

Inserting the closed form $\partial\phi_{(n)}/\partial p - 1 = 1 + pf'/f + (n-1)pf/F$ on the left and invoking the FOC identity $(n-1)(pf/F)nF^n = (n-1)S$, the resulting $(n-1)S \ln(1/F)$ terms cancel against the corresponding portion of $Sn \ln(1/F)$ on the right, leaving

$$\left[1 + p \frac{f'}{f} \right] nF^n \ln(1/F) > S(\ln(1/F) - 1).$$

Finally, substituting $S = npfF^{n-1}$ on the right and dividing by $nF^n > 0$ produces the criterion

$$\ln(1/F) \left[1 + p \frac{f'}{f} \right] > [\ln(1/F) - 1] \frac{pf}{F}.$$

For the standard log-concave families — uniform, exponential, Weibull with shape $k \geq 1$, power distributions with exponent $\alpha \geq 1$, truncated normal, and Beta with $a, b \geq 1$ — the inequality holds throughout the interior of the support, and hence at every FOC-determined p^{**} irrespective of n . For uniform and power distributions the coefficient of $\ln(1/F)$ vanishes, so the condition collapses to the trivial bound $pf/F > 0$; for exponential and Weibull it reduces to a single-parameter inequality in $t = (p/\lambda)^k$ that can be verified directly, and for normal and Beta numerical evaluation across the full support confirms strict inequality. Consequently $dS^{**}/dn > 0$ holds at $n = 2$ and for all larger n in each of these families; whether log-concavity of f alone suffices for the unconditional bound in full generality remains open.

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